A Partition of Unity Finite Element Method for heterogeneous media in underwater acoustics

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Abstract

Most of the physical environments of interest in underwater acoustics involve heterogeneous media. The spatial variability of these media depends on quantities such as temperature, salinity, water depth or the presence of biological components. The linear model which governs the time-harmonic propagation of acoustic waves in compressible inviscid fluids is given by the Helmholtz type equation

$$-\frac{\omega^2}{\rho c^2} p - \operatorname{div}\left(\frac{1}{\rho} \operatorname{grad} p\right) = 0,$$

where p is the Lagrangian fluctuation of the pressure field, ω is the angular frequency, ρ is the mass density, and c is the sound speed in the fluid. In heterogeneous media, both mass density and sound speed depend on the spatial position, either prescribed by piecewise-constant profiles (in stratified media) or given by smooth response functions.

The numerical approximation of this time-harmonic model can be obtained by means of Galerkin methods based on a polynomial local basis (such as Finite Element Methods). However, for high frequency values and even assuming homogeneous media (i.e., if ρ and c are constant), those discretizations lead to approximated solutions that differ from the exact solution due to the so-called pollution effect [1]. The accuracy and reliability of Helmholtz numerical approximations are based on pollution-free discrete methods, which should have a robust behavior with respect to the wavenumber. The Partition of Unity Finite Element Method (PUFEM) [5] has been considered among these pollution-free methods. Computational advantages and implementation drawbacks of the PUFEM discretization have been studied numerically (see [6], [3] and [4]). Error estimates for Finite Element solutions of the one-dimensional Helmholtz equation in homogeneous media have been already studied by Babŭska and Ihlenburg in [2].

In this work, *a priori* one-dimensional error estimates are derived for PUFEM in homogeneous media, where plane waves are used to modify the discretization space. The approximability of the exact solution in such discrete space is deduced from some interpolation estimates involving only exponential-type basis functions. Error estimates for PUFEM are obtained in terms of the wavenumber on the Helmholtz equation, the mesh size and an additional perturbation parameter introduced in the wavenumbers of the basis functions of the discrete PUFEM space.

The time-harmonic acoustic heterogeneous model described above is numerically (both in one and two dimensions) solved by means of a PUFEM method which uses a novel enrichment procedure. Instead of using simple plane wave solutions of a constant-coefficient partial differential equation, the proposed method is based on local solutions of the variable coefficient model.

Some numerical experiments are carried out to illustrate the performance of this novel PUFEM procedure with respect to the mesh size, the quadrature schemes (either exact [7] or semi-analytic schemes based on high-order Gauss-Legendre rules), the number of functions used in the enrichment procedure, and the behavior of the discrete method with respect to the angular frequency.

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Bibliography

[1] A. Deraemaeker, I. Babŭska, P. Bouillard, *Dispersion and pollution of the FEM solution for the Helmholtz equation in one, two and three dimensions*, International Journal for Numerical Methods in Engineering 46, 471-499 (1999).

[2] F. Ihlenburg, I. Babŭska, *Finite element solution of the Helmholtz equation with high wave number. Part I: The h-version of the FEM*, Computers and Mathematics with Applications 30, 9-37 (1995).

[3] O. Laghrouche, P. Bettess, E. Perrey-Debain, J. Trevelyan, *Wave interpolation finite elements for Helmholtz problems with jumps in the wave speed*, Computer Methods in Applied Mechanics and Engineering 194, 367-381 (2005).

[4] O. Laghrouche, P. Bettess, E. Perrey-Debain, J. Trevelyan, *Extension of the PUFEM to elastic wave propagation in layered media*, Journal of Computational Acoustics 20, 1240006 (2012).

[5] J. M. Melenk, I. Babŭska, *The partition of unity finite element method: Basic theory and applications*, Journal of Computer Methods in Applied Mechanics and Engineering 139, 289-314 (1996).

[6] J. M. Melenk, *On generalized finite element method*, Ph.D. Thesis, University of Maryland, (1995).

[7] P. Ortiz , E. Sánchez, *An improved partition of unity finite element model for diffraction problems*, International Journal for Numerical Methods in Engineering 50, 2727-2740 (2001).