Representation Learning Based on Givens Transformation and its applications

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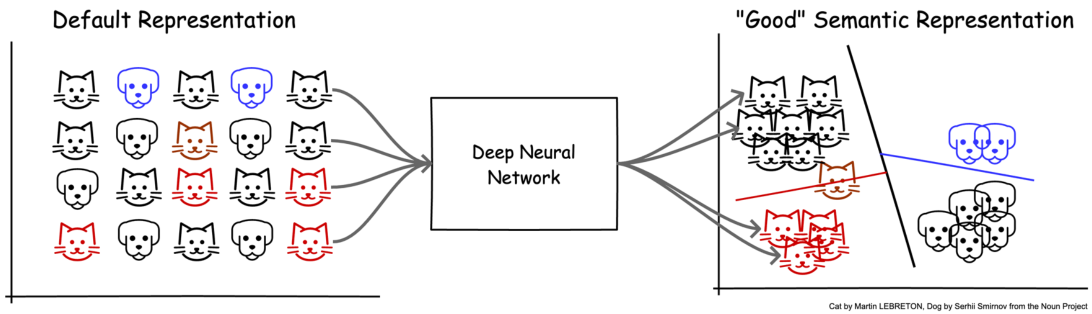
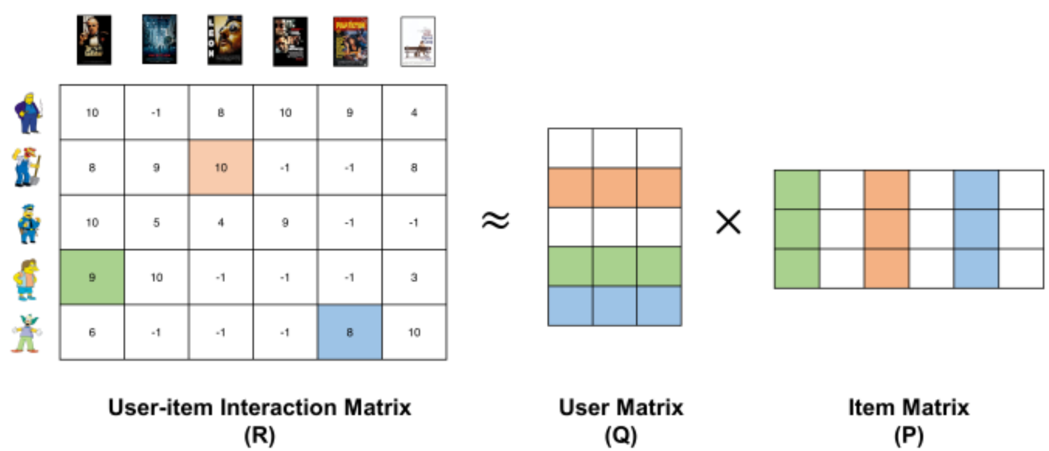
1 Background

- Representation Learning (RL) involves automatically finding the **features or representations of data** useful for predictive tasks.



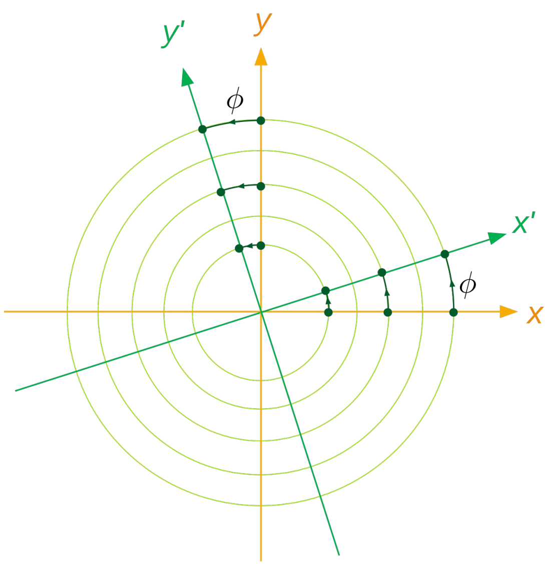
- Matrix Factorization (MF) is a technique to decompose a matrix into a product of matrices, revealing **hidden structures** in data.
- By decomposing data into simpler matrices, it reveals the **underlying structure** or patterns that are essential for representation learning.

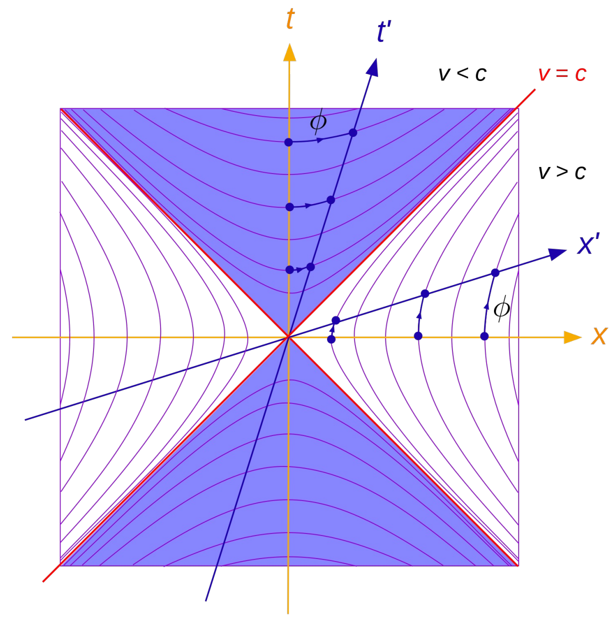


1 Background

- The **“Orthogonal constraints”** of matrix decomposition theories.

- **Advantages**:
  - Ensures the **independence of latent dimensions**;
  - removes **redundant information**;
  - improves the efficiency of representation learning;

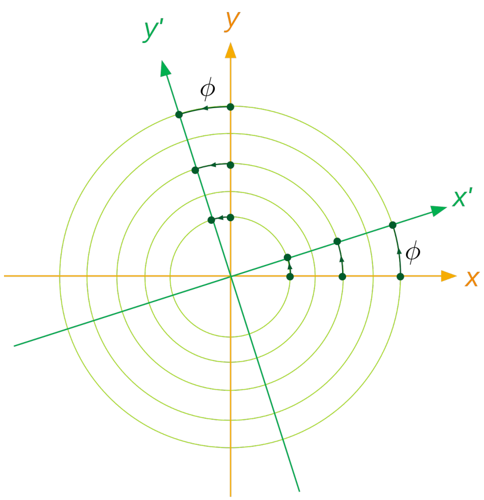


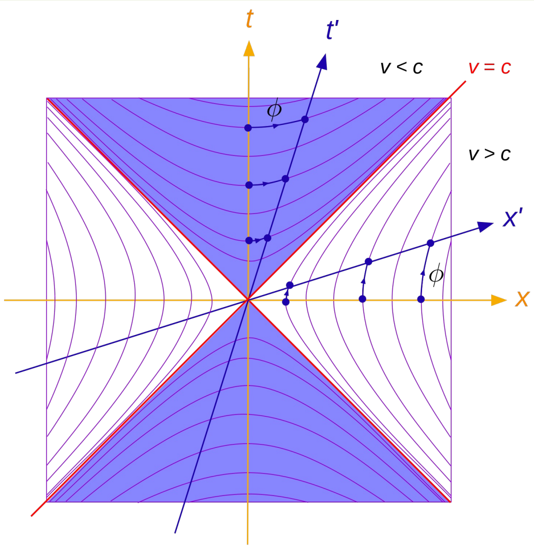


1 Background

- The **“Orthogonal constraints”** of matrix decomposition theories

- **Advantages**:
  - Ensures the **independence of latent dimensions**;
  - removes **redundant information**;
  - improves the efficiency of representation learning;
- **Disadvantages:**
  - When manipulating representations, the orthogonality of the representation space can be easily **disrupted**, affecting the effectiveness of subsequent models;
  - Naturally introduces constraints, thereby **increasing** the number of representational **elements**.





1 Background

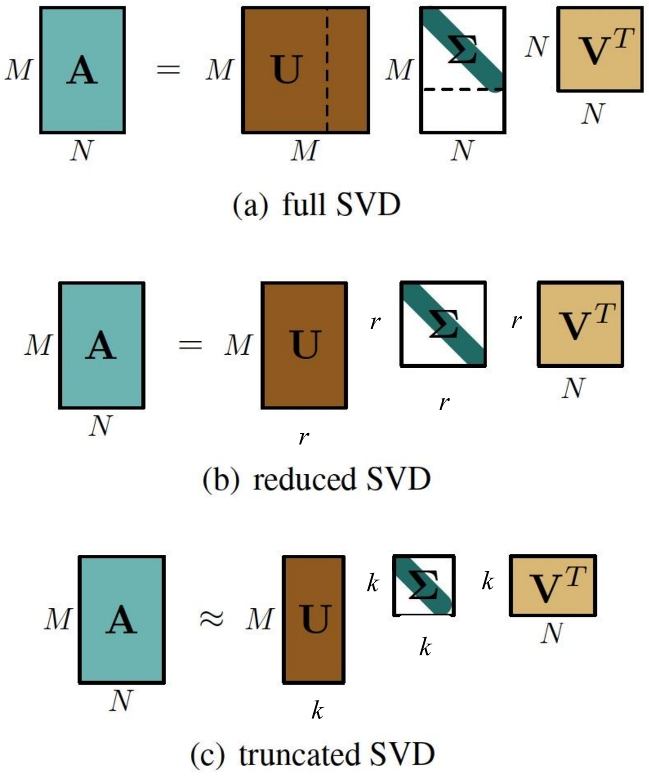
- The “**Orthogonal constraints**” of matrix decomposition theories
- Typical Example: Singular Value Decomposition, SVD

✓ ***U and V*** **have inherent constraints，say** $\boldsymbol{U}$**=**

$[\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_\mathrm{r}]$**:**

$$\boldsymbol{u}_\mathrm{i}\boldsymbol{u}_j = \begin{cases} \mathbf{1}, \boldsymbol{i} = \boldsymbol{j} \\ \mathbf{0}, \boldsymbol{i} \neq \boldsymbol{j} \end{cases}$$

✓ **For column orthogonal matrices with** $\boldsymbol{r}$ **columns, normality introduces** $\boldsymbol{r}$ **constraints, orthogonality introduces** $\boldsymbol{r}(\boldsymbol{r}-\mathbf{1})/\mathbf{2}$ **constraints, thus** $\boldsymbol{r}(\boldsymbol{r}+\mathbf{1})/\mathbf{2}$ **constraints in total.**



(a) full SVD
(b) reduced SVD
(c) truncated SVD

2 Methodology applied

- In order to solve existed problems, we are going to use Givens Transformation to construct the representation space.

- For arbitrary vector $W = (w_1, w_2, \dots, w_m)^T$, $1 \le k < i \le m$, Givens matrix is defined as

$$G_{ki} = \begin{bmatrix} I_{k-1} & 0 & \dots & 0 & 0 \\ 0 & c & \dots & d & 0 \\ \dots & \dots & I_{i-k-1} & \dots & \dots \\ 0 & -d & \dots & c & 0 \\ 0 & 0 & \dots & 0 & I_{m-i} \end{bmatrix}$$

where, $c = \frac{w_k}{s_{ki}} = cos\theta_{ki}, d = \frac{w_i}{s_{ki}} = sin\theta_{ki}$, $(s_{ki} = (w_k^2 + w_i^2)^{1/2})$ posed in (k, i) and (i, k).

- $G_{ki}W$ represents a counterclockwise rotation of the vector x in the (i, j) plane of θ radians：

$$G_{ki}W = (w_1, \dots, w_{k-1}, s_{ki}, \dots, w_{i-1}, 0, \dots)^T;$$

- Thus $G_{1m}G_{1(m-1)} \dots G_{12}W = (s, 0, \dots, 0)^T$, where $s = (w_1^2 + w_2^2 + \dots + w_m^2)^{1/2}$.

- For normalized vector and $k = 1$：

$$G_{1m}G_{1(m-1)} \dots G_{12}W = (1, 0, \dots, 0)^T.$$

2 Methodology applied

- **Givens Transformation of Column Orthogonal Matrices**
- **For column orthogonal matrix $A \in \mathbb{R}^{m\times r}$, there are$[(m\times r) - r(r+1)/2]$Givens matrices(k = 1,2,…,r; i = k+1, k+2,…,m) that satisfy：**

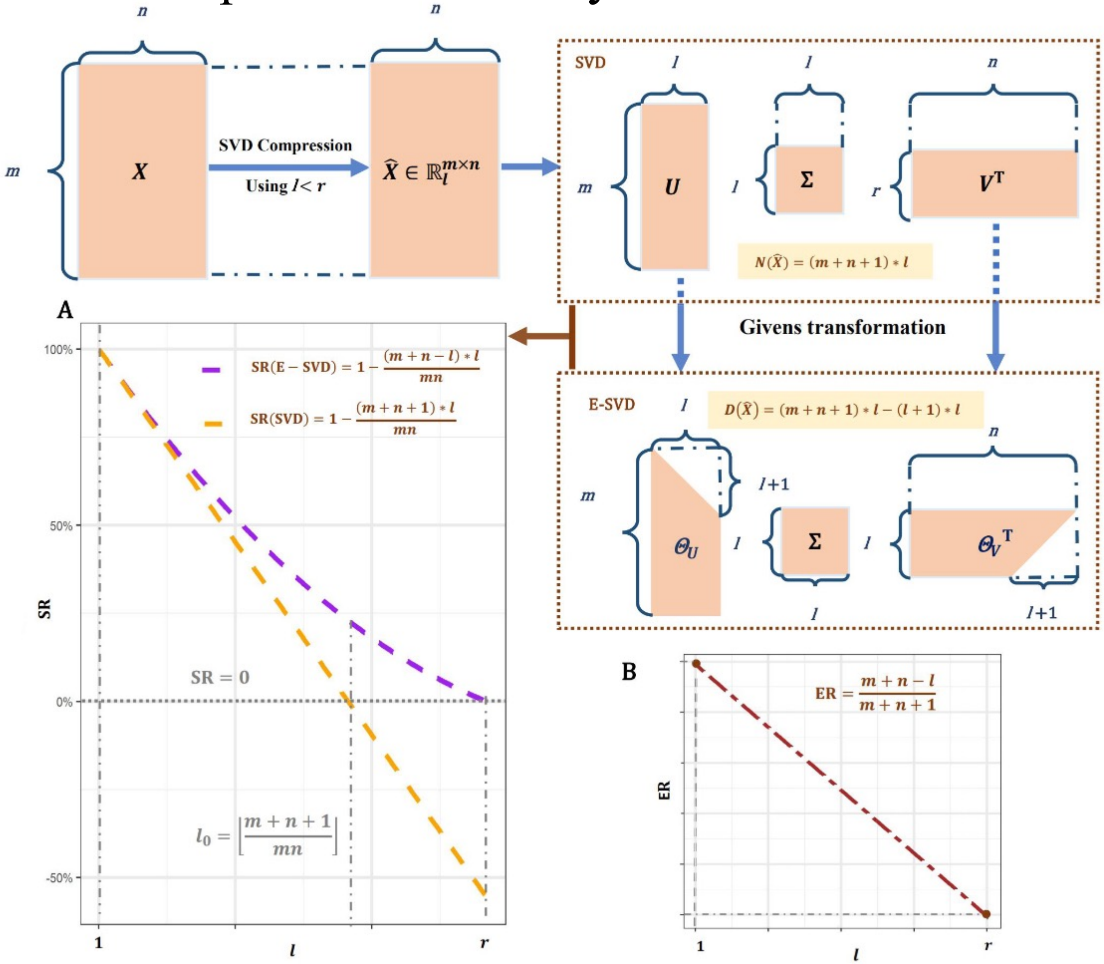
$$(G_{rm}G_{r(m-1)}\ldots G_{r(r+1)})\ldots(G_{2m}G_{2(m-1)}\ldots G_{23})(G_{1m}G_{1(m-1)}\ldots G_{12})A = \begin{bmatrix} I_r \\ O_{(m-r)\times r} \end{bmatrix},$$

- **Inverse Transformation of Column Orthogonal Matrices**
- **Storing $[(m\times r) - r(r+1)/2]$Givens matrices，we can get $A \in \mathbb{R}^{m\times r}$ ：**

$$A = (G_{12}^{\mathrm{T}}G_{13}^{\mathrm{T}}\ldots G_{1m}^{\mathrm{T}})\ldots(G_{r(r+1)}^{\mathrm{T}}G_{r(r+2)}^{\mathrm{T}}\ldots G_{rm}^{\mathrm{T}})\begin{bmatrix} I_r \\ O_{(m-r)\times r} \end{bmatrix}.$$

3 Constructing embedding space

- Based on that, we can easily acquire an enhanced version of SVD (**E-SVD**), a method eliminating all redundancies in SVD matrices and enhancing SVD compression losslessly.



- (1) The number of storage units SVD compression:

$$(m+n+1)\cdot l;$$

- (2) The freely valued elements of orthonormal column matrix $\boldsymbol{U}$:

$$m\cdot l-0.5^{*}\, l\cdot(l+1);$$

- (3) The freely valued elements of orthonormal column matrix $\boldsymbol{V}$:

$$n\cdot l-0.5^{*}\, l\cdot(l+1);$$

- (4) The nonzero elements of diagonal matrix $\boldsymbol{\Sigma}$: l;

- (5) The number of storage units after E-SVD:

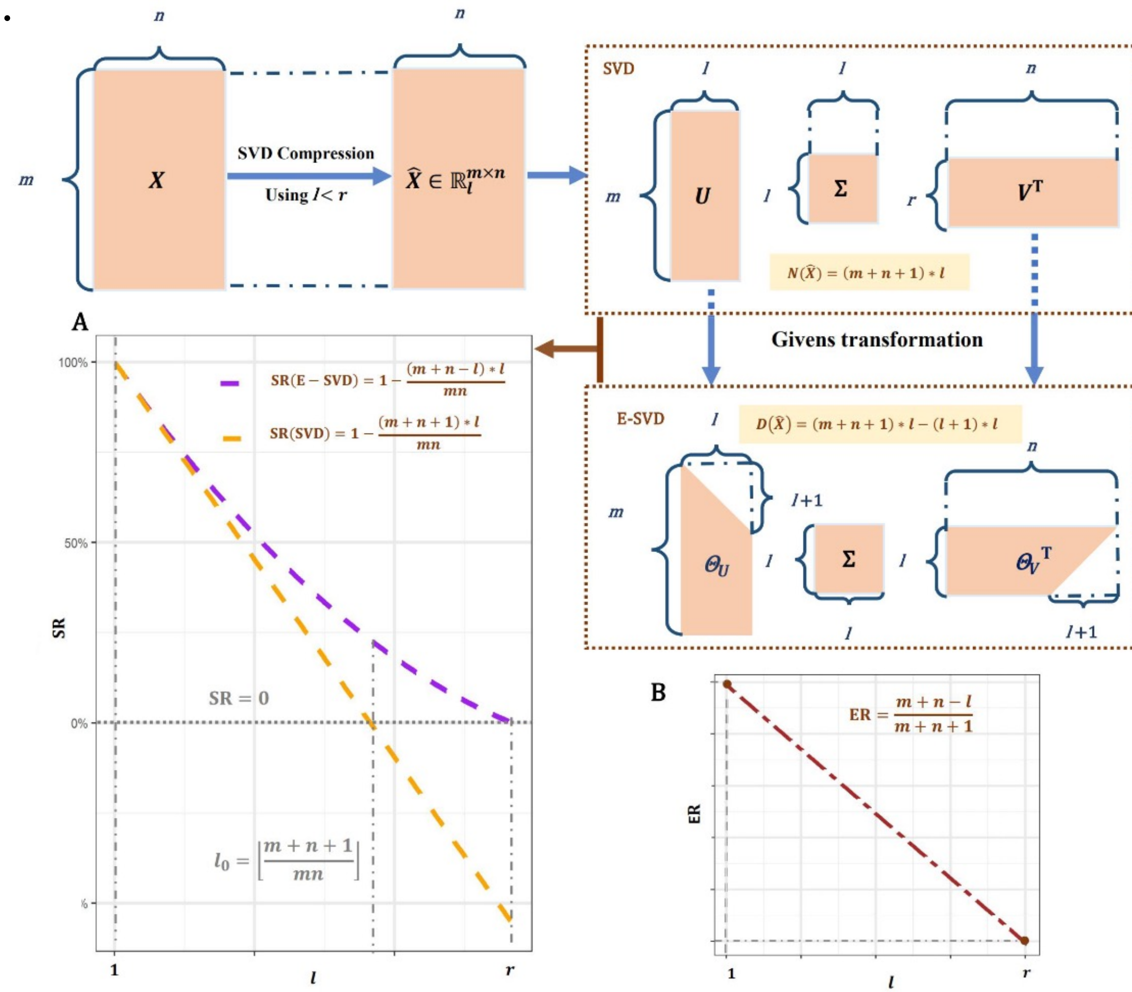
$$(m+n-l)\cdot l.$$

[1] Wang H, Zhang Y, Zhao J. Enhancing the SVD compression losslessly[J]. Journal of Computational Science, 2023, 74: 102182.

3 Constructing embedding space

Storage ratio analysis

- We can use Storage Ratio (SR) to indicate the ratio of the reduced storage units to that of the original matrix.



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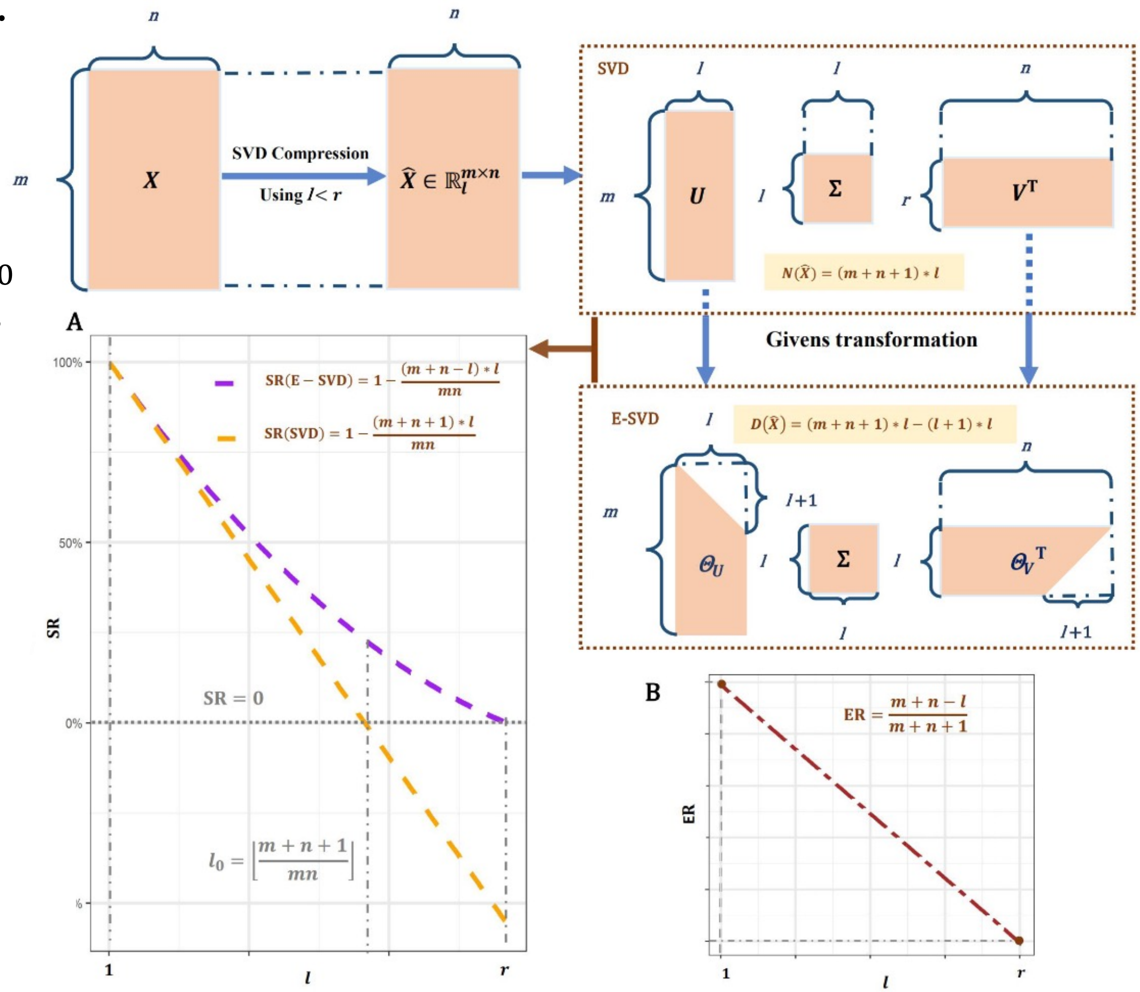
✓ The SVD compression would fail to compress data in some situations.

✓ To clearer refer to the limitation of SVD compression, we would denote l_0 as the l where SVD compression fails, which satisfies $l_0 = \lfloor \frac{m \cdot n}{m+n+1} \rfloor$.



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✓ **When SVD fails, E-SVD can still compress the data and use only 75% storage space to preserve the same amount of information.**

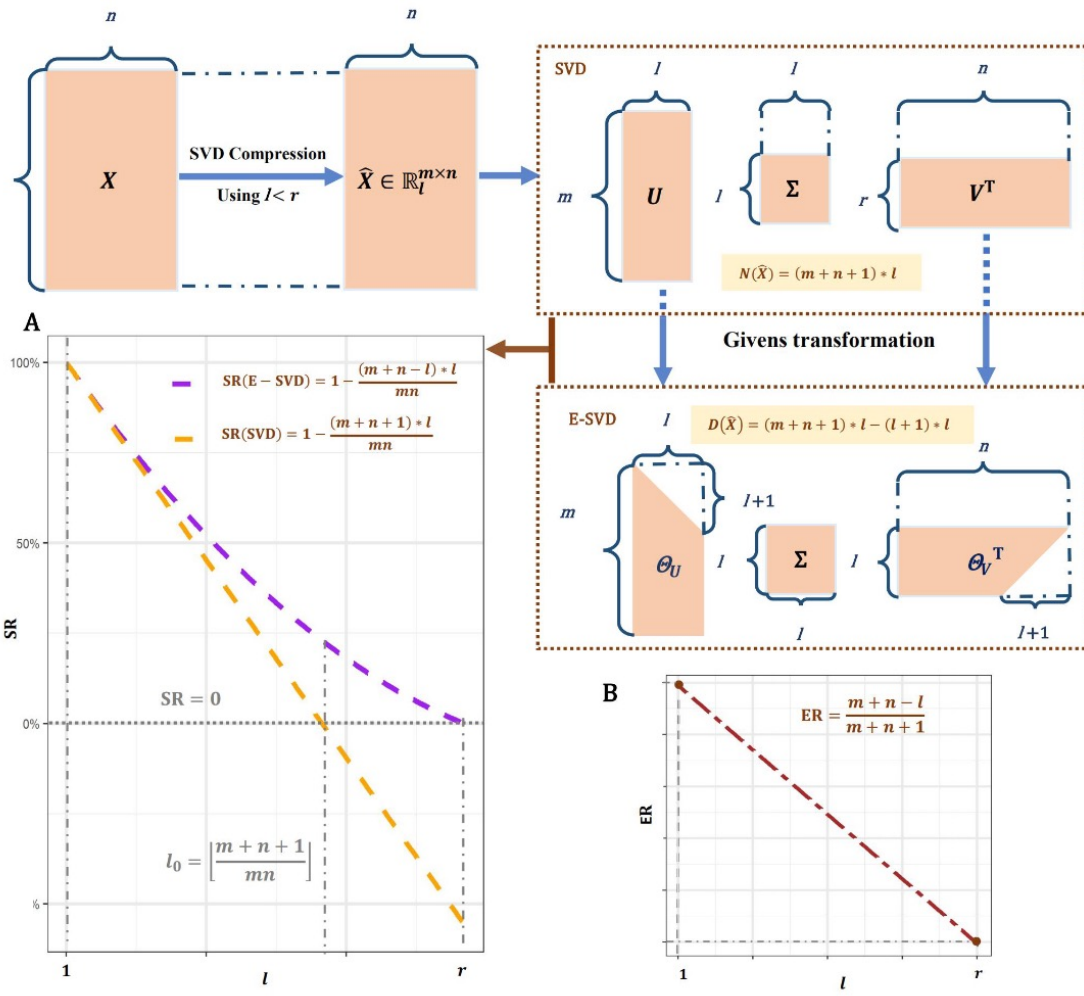
Proof. For $X \in \mathbb{R}_r^{m \times n}$, when $m = n$ and $l = \frac{m \cdot n}{m+n+1}$,

$$\lim_{m \to +\infty} \text{ER} = \lim_{m \to +\infty} [\frac{2m - \frac{m^2}{2m+1}}{2m+1}]$$

$$= \lim_{m \to +\infty} [\frac{2m}{2m+1} - \frac{m^2}{(2m+1)^2}].$$

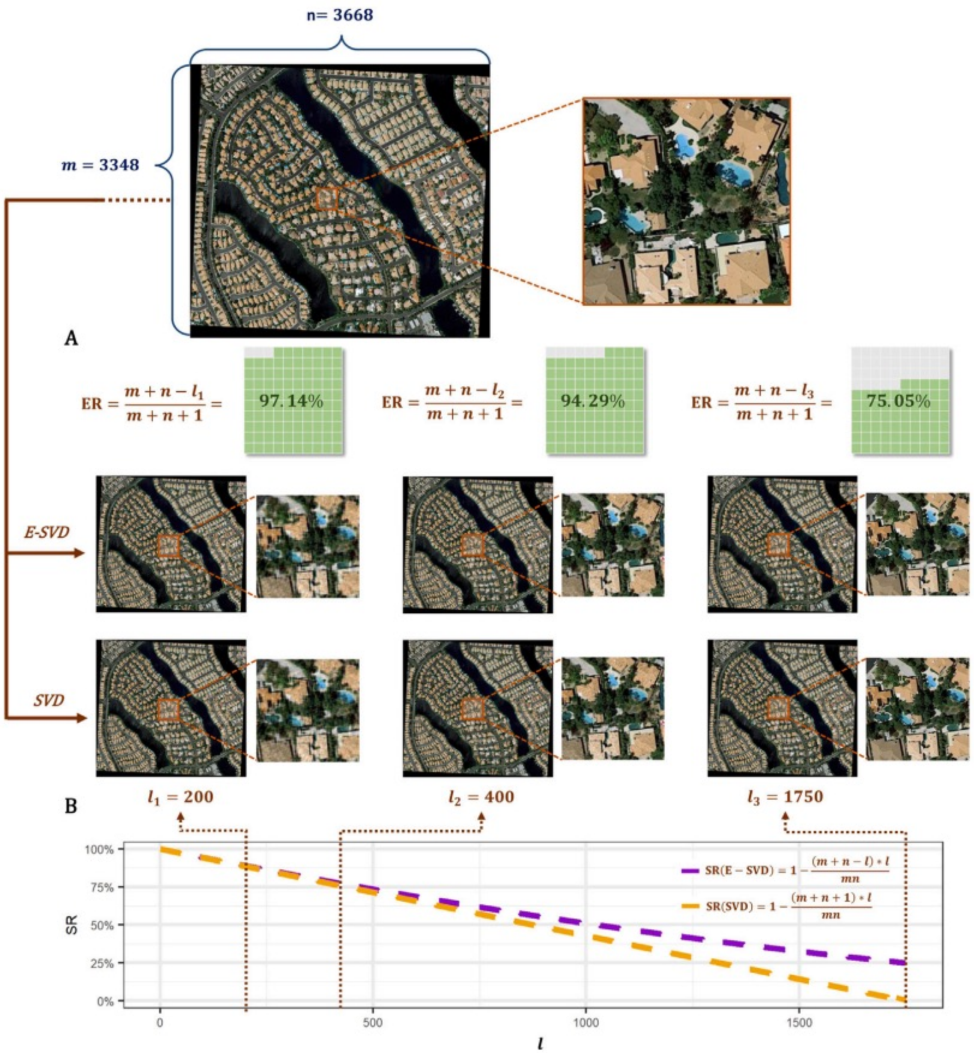
Using L'Hospital's rule, we can get

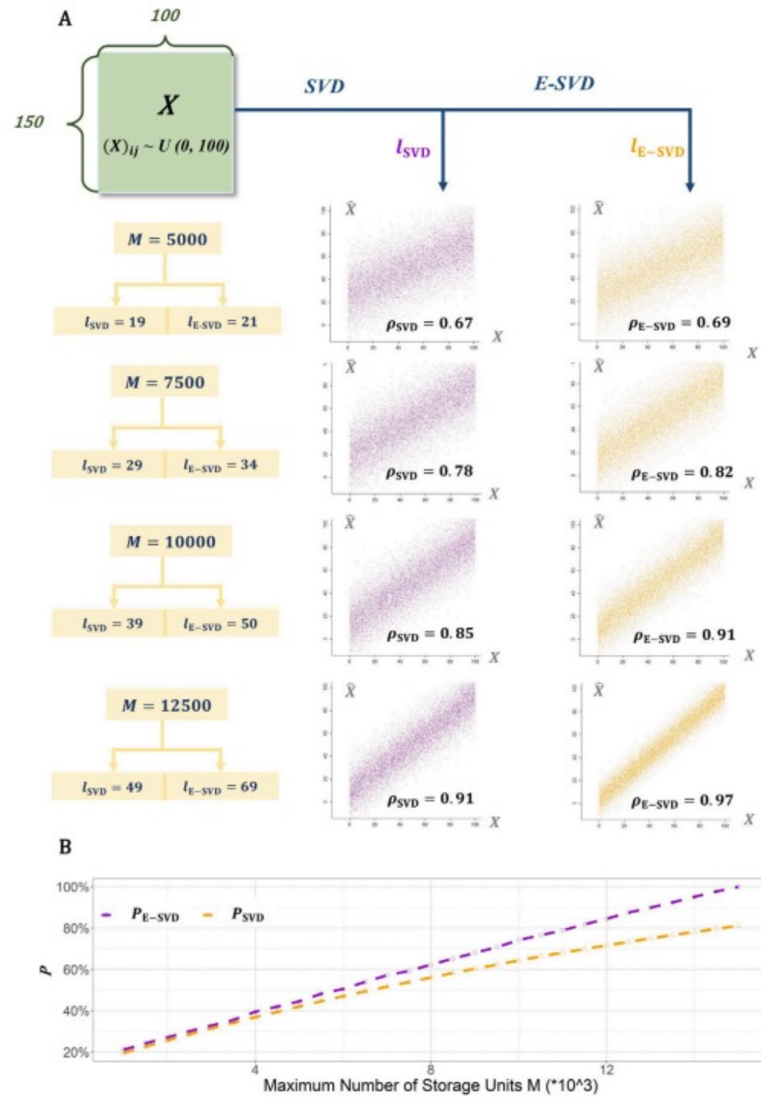
$$\lim_{m \to +\infty} \text{ER} = 1 - \frac{2m}{4(2m+1)} = 1 - \frac{1}{4} = \frac{3}{4} = 75\%.$$



3 Constructing embedding space

Experimental Evidences





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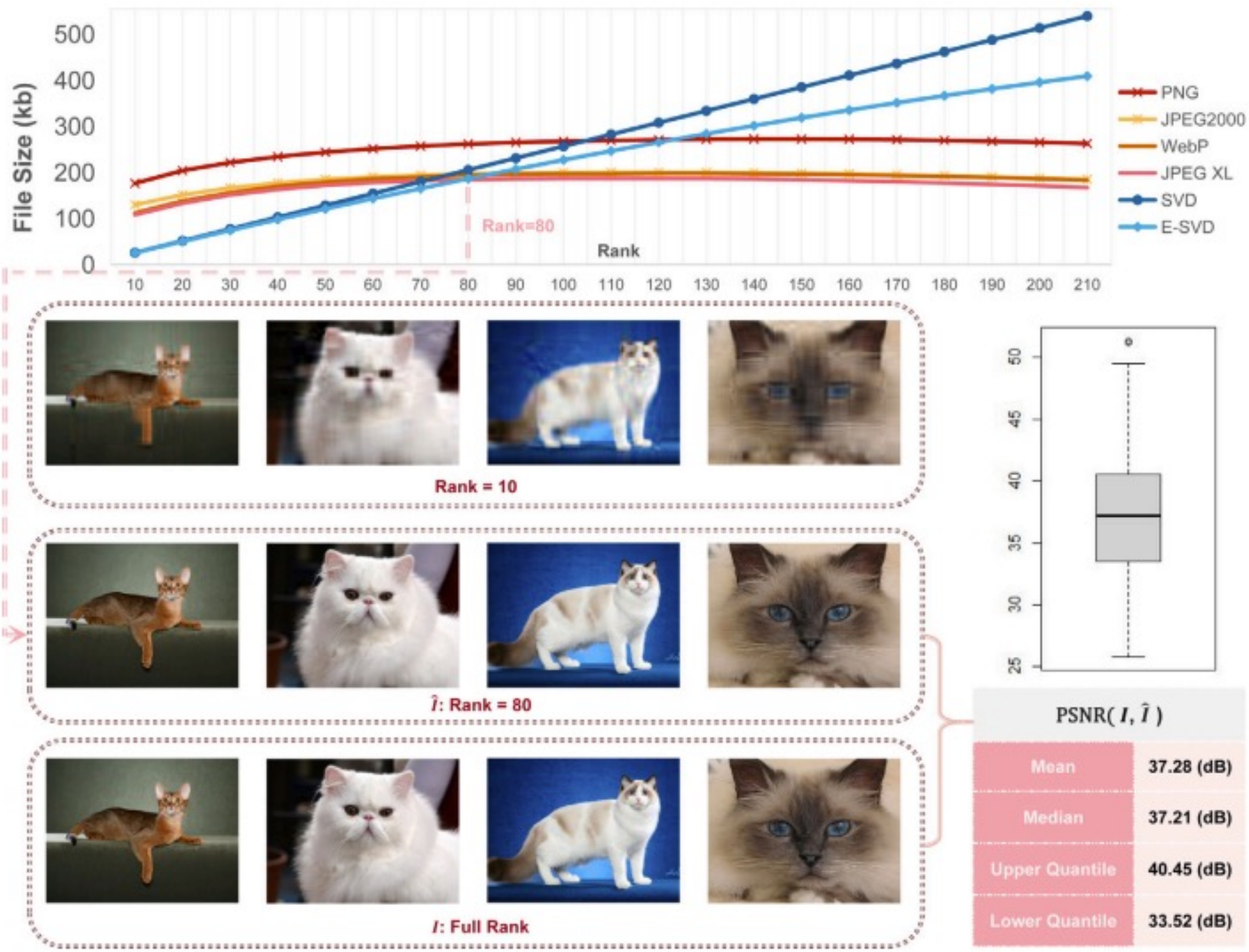
3 Constructing embedding space

Experimental Evidences



PSNR(I, Î)	
Mean	37.28 (dB)
Median	37.21 (dB)
Upper Quantile	40.45 (dB)
Lower Quantile	33.52 (dB)

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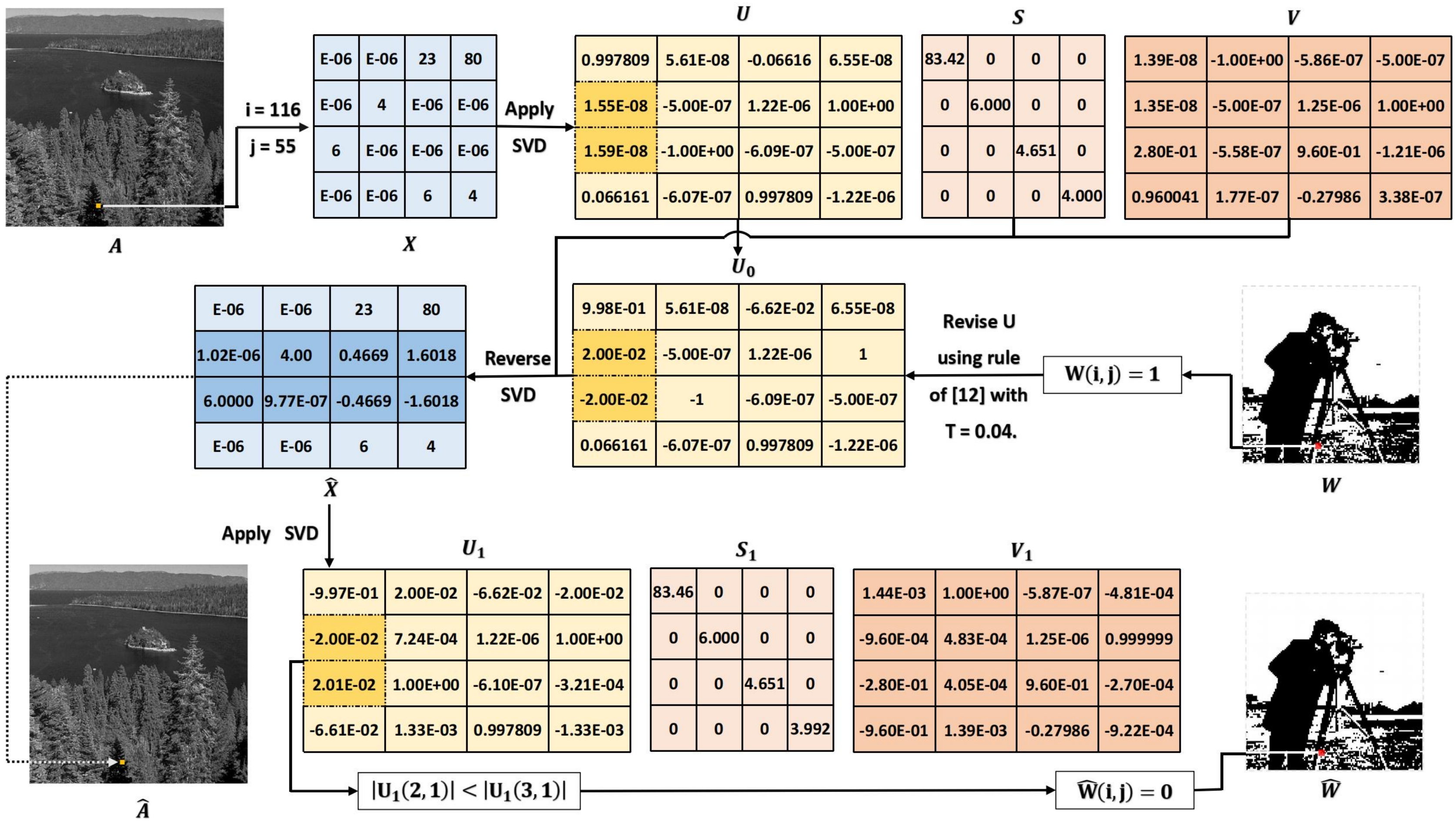
4 The invertibility of embedding

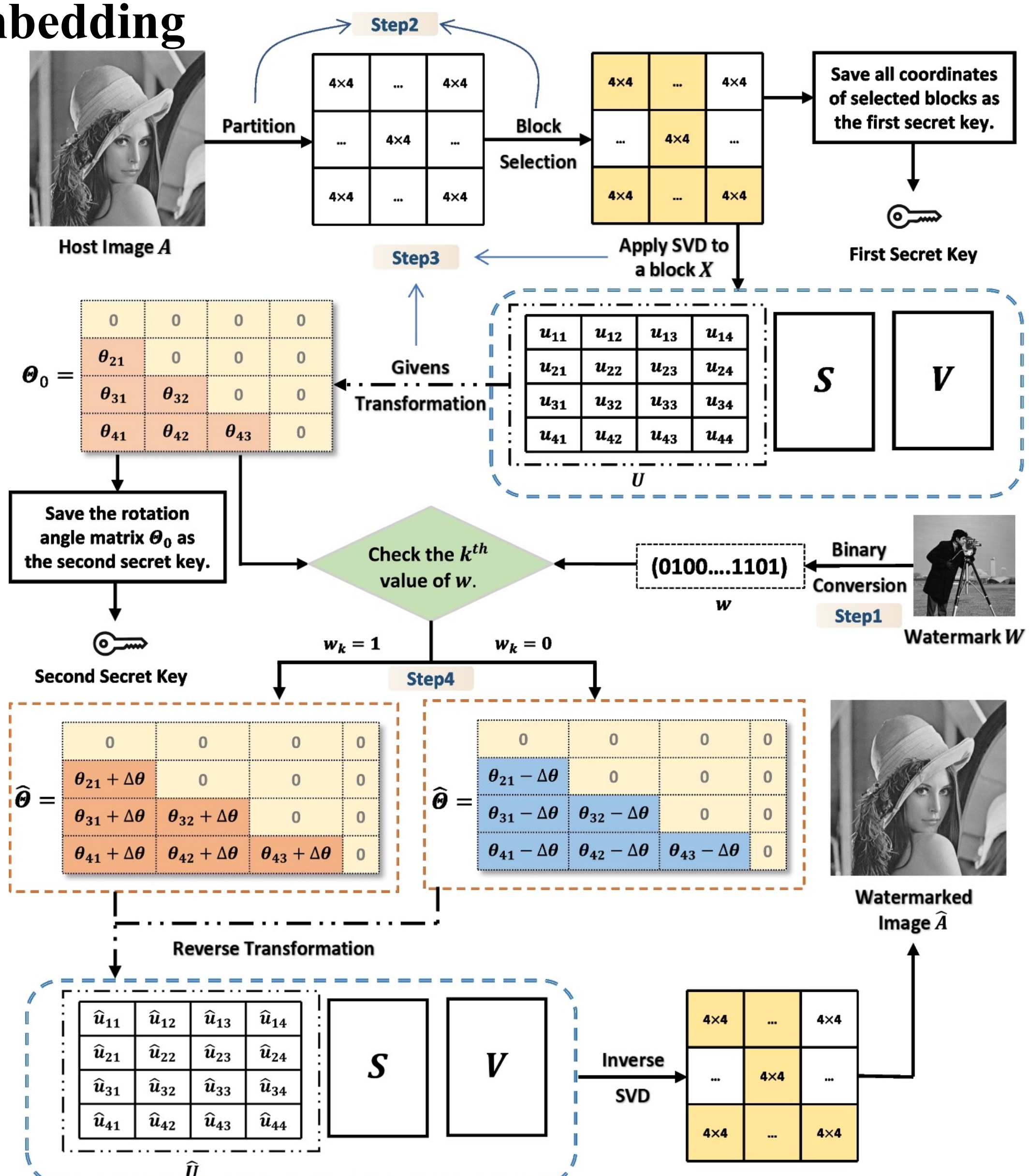
- Origin from a problem in digital watermarking:



[2] Zhang Y, Wang H, Zhao J. Eliminating orthonormal constraints of SVD to guarantee full retrievability of blind watermarking[J]. Multimedia Tools and Applications, 2023: 1-27.

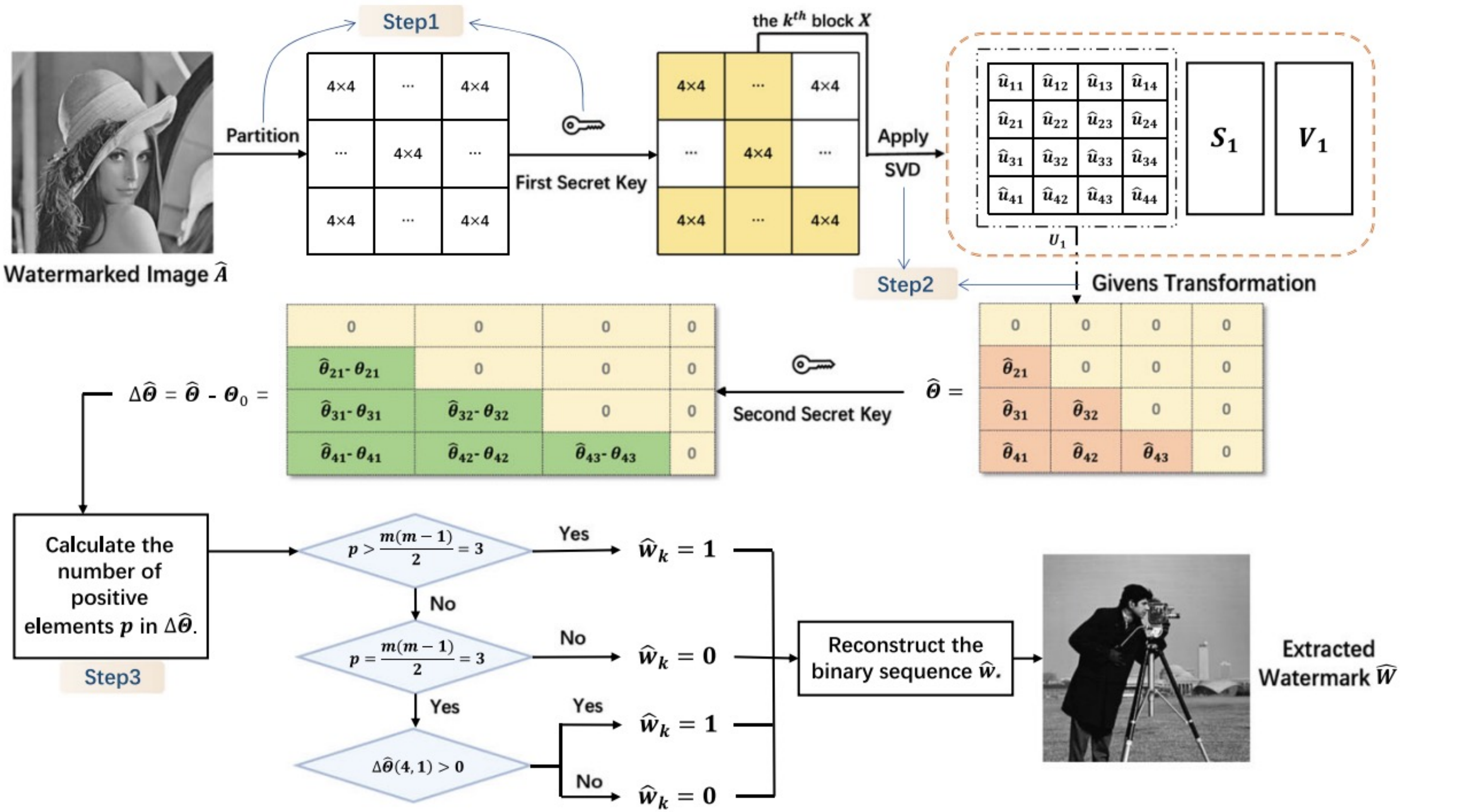
4 The invertibility of embedding

Watermark Embedding



4 The invertibility of embedding

Watermark Extraction



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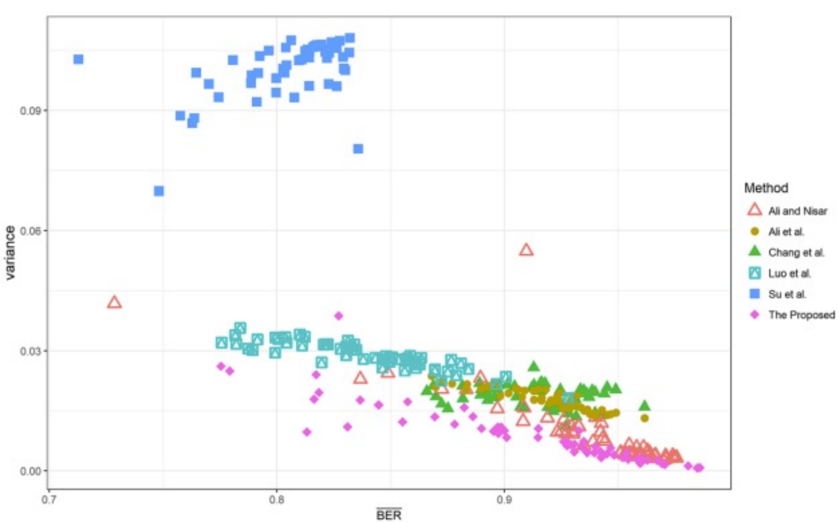
4 The invertibility of embedding

Experiment results – Robustness comparison

	Subbands	Modifying singular values	Modifying singular vectors
Ali et al.	LL of RIDWT		✓
Ali and Nisar	LL of DWT	✓	
Chang et al.	\		✓
Luo et al.	LL of DWT		✓
Su et al.	\	✓	
Proposed	\		✓



Attack type	Ali and Nisar	Ali et al.	Chang et al.	Luo et al.	Su et al.	Proposed
Gaussian Noise	0.9999	0.9999	0.9859	0.9991	0.9968	0.9958
S&P Noise	0.9998	0.9999	0.9957	0.9994	0.9967	0.9959
Speckle Noise	0.9999	1.0000	0.9957	0.9995	0.9967	0.9961
Average Filter	0.8858	0.7429	0.7377	0.6254	0.7482	0.7698
Gaussian Low-Pass Filter	0.9840	0.9624	0.9545	0.8381	0.9694	0.9475
JP2K CR=2	0.9949	0.9976	0.9926	0.9713	0.9968	0.9818
JP2K CR=4	0.9184	0.9620	0.9472	0.8373	0.9782	0.8828
JP2K CR=6	0.7827	0.8972	0.8768	0.7153	0.9266	0.7810
JPEG QF=50	0.7752	0.7423	0.7176	0.5726	0.9761	0.8259
JPEG QF=70	0.9041	0.8463	0.8315	0.6380	0.9932	0.8890
JPEG QF=90	0.9909	0.9771	0.9697	0.8309	0.9965	0.9574
Brighten	0.9996	0.9999	0.9958	0.9955	0.0198	0.9413
Darken	0.9043	0.9776	0.9556	0.9632	0.0224	0.9147

[2] Zhang Y, Wang H, Zhao J. Eliminating orthonormal constraints of SVD to guarantee full retrievability of blind watermarking[J]. Multimedia Tools and Applications, 2023: 1-27.

5 The sensitivity of embedding

- If the matrix is an image, are there any connection between the change of an image and the change of corresponding θ?

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$$U = G_{21}^{T}G_{31}^{T}G_{41}^{T}G_{32}^{T}G_{42}^{T}G_{43}^{T}I_4$$

$$= \begin{bmatrix} cos\theta_{21} & -sin\theta_{21} & 0 & 0 \\ sin\theta_{21} & cos\theta_{21} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta_{31} & 0 & -sin\theta_{31} & 0 \\ 0 & 1 & 0 & 0 \\ sin\theta_{31} & 0 & cos\theta_{31} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} cos\theta_{41} & 0 & 0 & -sin\theta_{41} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ sin\theta_{41} & 0 & 0 & cos\theta_{41} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\theta_{32} & -sin\theta_{32} & 0 \\ 0 & sin\theta_{32} & cos\theta_{32} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\theta_{42} & 0 & -sin\theta_{42} \\ 0 & 0 & 1 & 0 \\ 0 & sin\theta_{42} & 0 & cos\theta_{42} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & cos\theta_{43} & -sin\theta_{43} \\ 0 & 0 & sin\theta_{43} & cos\theta_{43} \end{bmatrix} I_4$$

$$= \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{bmatrix},$$

Recall our purpose that to detect image change quickly, it is reasonable to use the 1-rank approximation of $\boldsymbol{a}$ as the analysis object, then we can reconstruct $\boldsymbol{a}$ using θ.

$$\hat{a}_{11} = \lambda_1 (cos\theta_{21}cos\theta_{31}cos\theta_{41})^{\boldsymbol{U}} (cos\theta_{21}cos\theta_{31}cos\theta_{41})^{\boldsymbol{V}},$$

$$\hat{a}_{21} = \lambda_1 (sin\theta_{21}cos\theta_{31}cos\theta_{41})^{\boldsymbol{U}} (cos\theta_{21}cos\theta_{31}cos\theta_{41})^{\boldsymbol{V}},$$

$$\hat{a}_{31} = \lambda_1 (sin\theta_{31}cos\theta_{41})^{\boldsymbol{U}} (cos\theta_{21}cos\theta_{31}cos\theta_{41})^{\boldsymbol{V}},$$

$$\hat{a}_{41} = \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (cos\theta_{21}cos\theta_{31}cos\theta_{41})^{\boldsymbol{V}},$$

In order to figure out that when $\boldsymbol{a}$ changes, which θ would be more indicative, it is naturally for us to think about calculating derivatives.

$$\begin{aligned} d(\hat{a}_{41}) = & \lambda_1 (cos\theta_{41})^{\boldsymbol{U}} (cos\theta_{21}cos\theta_{31}cos\theta_{41})^{\boldsymbol{V}} \\ & \cdot d(\theta_{41}^{\boldsymbol{U}}) + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (-sin\theta_{21}cos\theta_{31}cos\theta_{41})^{\boldsymbol{V}} \\ & \cdot d(\theta_{21}^{\boldsymbol{V}}) + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (-cos\theta_{21}sin\theta_{31}cos\theta_{41})^{\boldsymbol{V}} \\ & \cdot d(\theta_{31}^{\boldsymbol{V}}) + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (-cos\theta_{21}cos\theta_{31}sin\theta_{41})^{\boldsymbol{V}} \cdot d(\theta_{41}^{\boldsymbol{V}}) \end{aligned}$$

[3] Zhang Y, Zhao J. θ is all you need: Revisiting SVD in caputuring changes in matrices[C]//Proceedings of the 2022 5th International Conference on Algorithms, Computing and Artificial Intelligence. 2022: 1-9.

5 The sensitivity of embedding

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Specifically, we can find out for some elements, their total differentials are not depend on all angles, i.e. only relates to specific angles. Typically, elements in the forth line satisfy.

$$
\begin{aligned}
d(\hat{a}_{41}) =& \lambda_1 (cos\theta_{41})^{\boldsymbol{U}} (cos\theta_{21} cos\theta_{31} cos\theta_{41})^{\boldsymbol{V}} \\
& \cdot d(\theta_{41}^{\boldsymbol{U}}) + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (-sin\theta_{21} cos\theta_{31} cos\theta_{41})^{\boldsymbol{V}} \\
& \cdot d(\theta_{21}^{\boldsymbol{V}}) + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (-cos\theta_{21} sin\theta_{31} cos\theta_{41})^{\boldsymbol{V}} \\
& \cdot d(\theta_{31}^{\boldsymbol{V}}) + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (-cos\theta_{21} cos\theta_{31} sin\theta_{41})^{\boldsymbol{V}} \cdot d(\theta_{41}^{\boldsymbol{V}}) \\
d(\hat{a}_{42}) =& \lambda_1 (cos\theta_{41})^{\boldsymbol{U}} (sin\theta_{21} cos\theta_{31} cos\theta_{41})^{\boldsymbol{V}} \\
& \cdot d(\theta_{41}^{\boldsymbol{U}}) + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (cos\theta_{21} cos\theta_{31} cos\theta_{41})^{\boldsymbol{V}} \\
& \cdot d(\theta_{21}^{\boldsymbol{V}}) + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (-sin\theta_{21} sin\theta_{31} cos\theta_{41})^{\boldsymbol{V}} \\
& \cdot d(\theta_{31}^{\boldsymbol{V}}) + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (-sin\theta_{21} cos\theta_{31} sin\theta_{41})^{\boldsymbol{V}} \cdot d(\theta_{41}^{\boldsymbol{V}}) \\
d(\hat{a}_{43}) =& \lambda_1 (cos\theta_{41})^{\boldsymbol{U}} (sin\theta_{31} cos\theta_{41})^{\boldsymbol{V}} \cdot d(\theta_{41}^{\boldsymbol{U}}) \\
& + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (cos\theta_{31} cos\theta_{41})^{\boldsymbol{V}} \cdot d(\theta_{31}^{\boldsymbol{V}}) \\
& + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (-sin\theta_{31} sin\theta_{41})^{\boldsymbol{V}} \cdot d(\theta_{41}^{\boldsymbol{V}}) \\
d(\hat{a}_{44}) =& \lambda_1 (cos\theta_{41})^{\boldsymbol{U}} (sin\theta_{41})^{\boldsymbol{V}} \cdot d(\theta_{41}^{\boldsymbol{U}}) \\
& + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (cos\theta_{41})^{\boldsymbol{V}} \cdot d(\theta_{41}^{\boldsymbol{V}}),
\end{aligned}
$$

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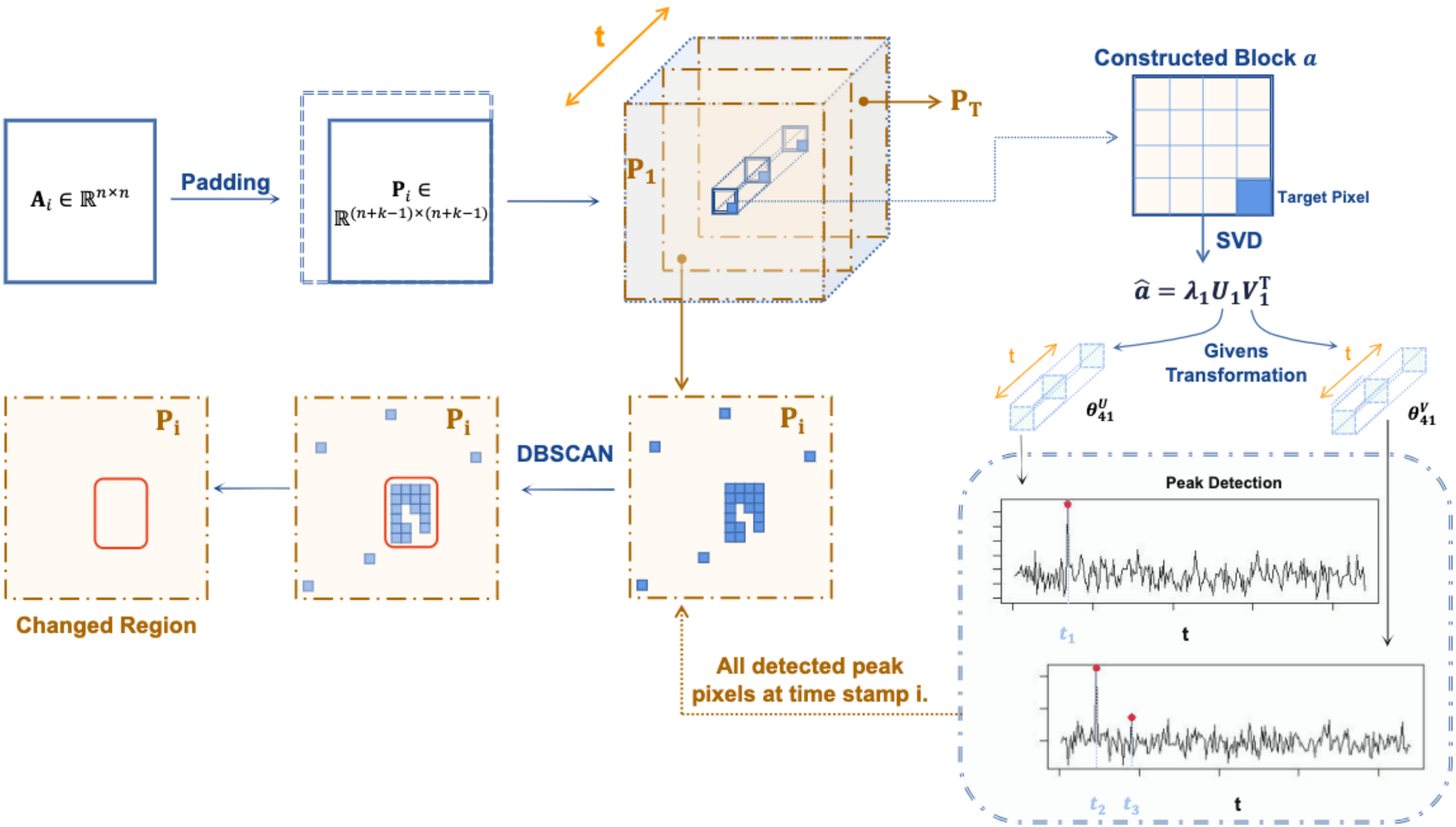
$$
\begin{aligned}
d(\hat{a}_{41}) = & \lambda_1 (cos\theta_{41})^{\boldsymbol{U}} (cos\theta_{21} cos\theta_{31} cos\theta_{41})^{\boldsymbol{V}} \\
& \cdot d(\theta_{41}^{\boldsymbol{U}}) + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (-sin\theta_{21} cos\theta_{31} cos\theta_{41})^{\boldsymbol{V}} \\
& \cdot d(\theta_{21}^{\boldsymbol{V}}) + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (-cos\theta_{21} sin\theta_{31} cos\theta_{41})^{\boldsymbol{V}} \\
& \cdot d(\theta_{31}^{\boldsymbol{V}}) + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (-cos\theta_{21} cos\theta_{31} sin\theta_{41})^{\boldsymbol{V}} \cdot d(\theta_{41}^{\boldsymbol{V}}) \\
d(\hat{a}_{42}) = & \lambda_1 (cos\theta_{41})^{\boldsymbol{U}} (sin\theta_{21} cos\theta_{31} cos\theta_{41})^{\boldsymbol{V}} \\
& \cdot d(\theta_{41}^{\boldsymbol{U}}) + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (cos\theta_{21} cos\theta_{31} cos\theta_{41})^{\boldsymbol{V}} \\
& \cdot d(\theta_{21}^{\boldsymbol{V}}) + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (-sin\theta_{21} sin\theta_{31} cos\theta_{41})^{\boldsymbol{V}} \\
& \cdot d(\theta_{31}^{\boldsymbol{V}}) + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (-sin\theta_{21} cos\theta_{31} sin\theta_{41})^{\boldsymbol{V}} \cdot d(\theta_{41}^{\boldsymbol{V}}) \\
d(\hat{a}_{43}) = & \lambda_1 (cos\theta_{41})^{\boldsymbol{U}} (sin\theta_{31} cos\theta_{41})^{\boldsymbol{V}} \cdot d(\theta_{41}^{\boldsymbol{U}}) \\
& + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (cos\theta_{31} cos\theta_{41})^{\boldsymbol{V}} \cdot d(\theta_{31}^{\boldsymbol{V}}) \\
& + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (-sin\theta_{31} sin\theta_{41})^{\boldsymbol{V}} \cdot d(\theta_{41}^{\boldsymbol{V}}) \\
d(\hat{a}_{44}) = & \lambda_1 (cos\theta_{41})^{\boldsymbol{U}} (sin\theta_{41})^{\boldsymbol{V}} \cdot d(\theta_{41}^{\boldsymbol{U}}) \\
& + \lambda_1 (sin\theta_{41})^{\boldsymbol{U}} (cos\theta_{41})^{\boldsymbol{V}} \cdot d(\theta_{41}^{\boldsymbol{V}}),
\end{aligned}
$$

It is evident that, among all elements in $\widehat{\boldsymbol{a}}$, $\widehat{\boldsymbol{a}}^{44}$ is the most "independent" one, which means that its perturbation can be completely captured by only two θ, i.e., $\theta_{41}^{\boldsymbol{U}}$ and $\theta_{41}^{\boldsymbol{V}}$.

5 The sensitivity of embedding

- Proposed change detection scheme



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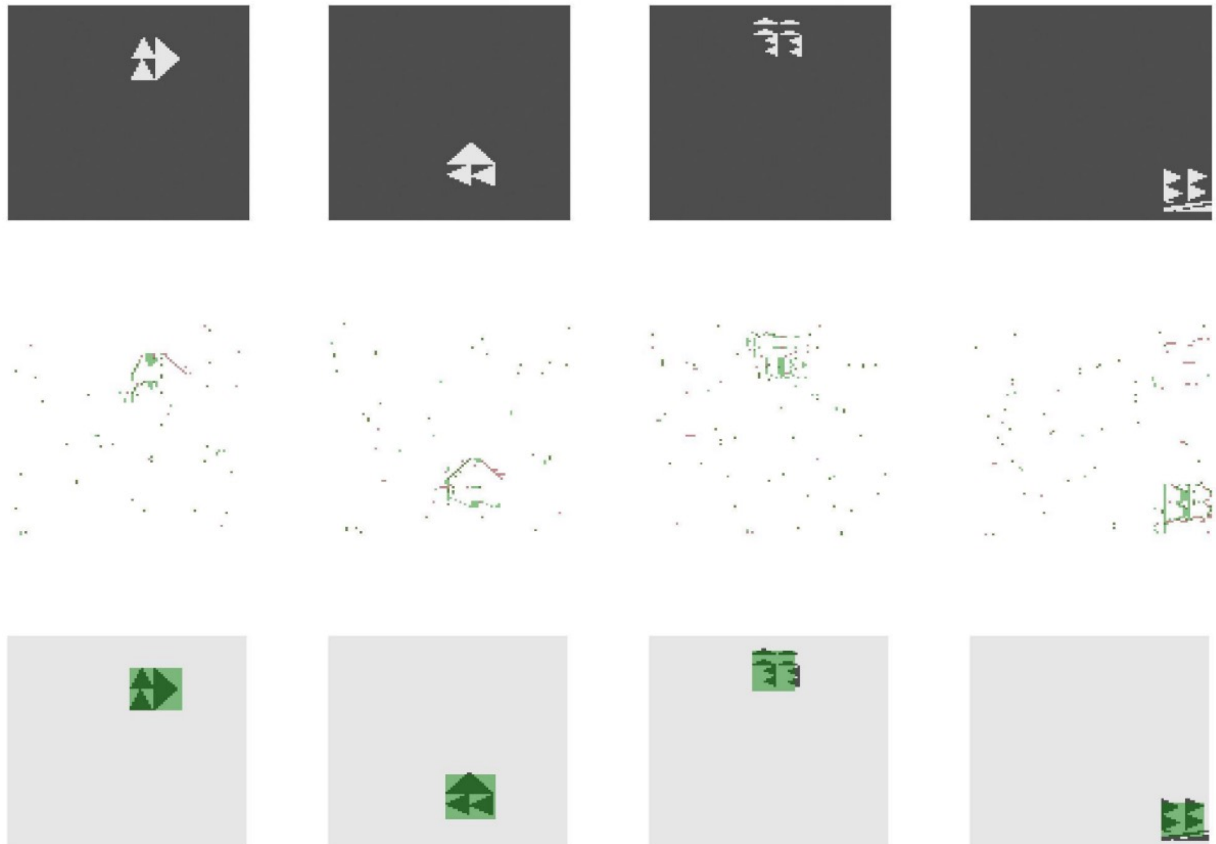
5 The sensitivity of embedding

- Simulation Result

Simulated Image

Detected changed pixels

Detected changed region

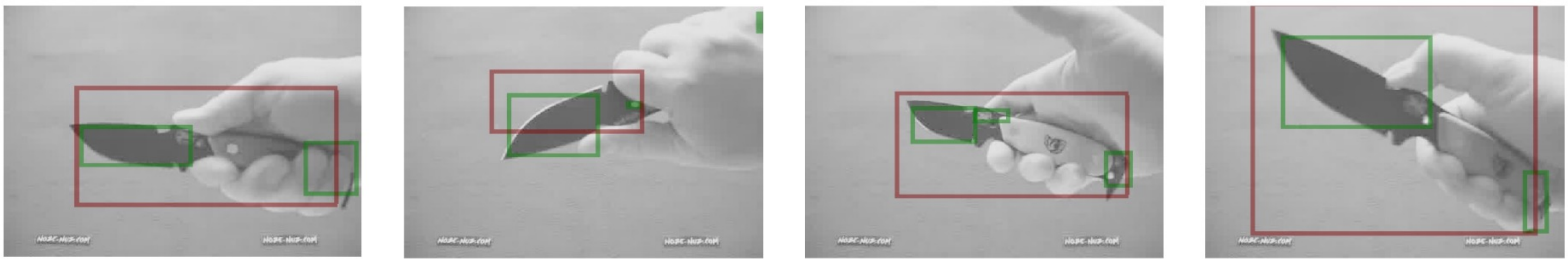


Median IOU	0.7960	0.8075	0.6192	0.6469

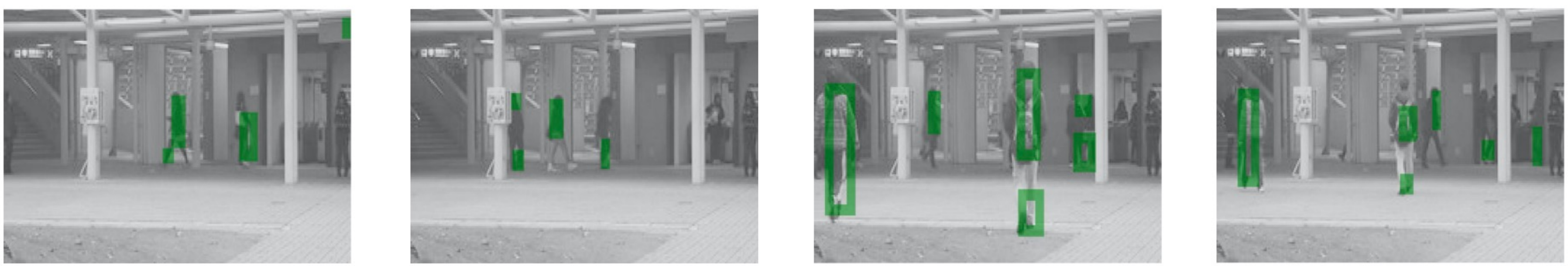
[3] Zhang Y, Zhao J. θ is all you need: Revisiting SVD in caputuring changes in matrices[C]//Proceedings of the 2022 5th International Conference on Algorithms, Computing and Artificial Intelligence. 2022: 1-9.

5 The sensitivity of embedding

- Real Result



(*a*) (*b*) (*c*) (*d*)



(*e*) (*f*) (*g*) (*h*)

[3] Zhang Y, Zhao J. θ is all you need: Revisiting SVD in caputuring changes in matrices[C]//Proceedings of the 2022 5th International Conference on Algorithms, Computing and Artificial Intelligence. 2022: 1-9.

6 Summary

Representation learning based on Givens transformation and its applications

- **The construction of embedding space**
  - **Theoretical Methodology**
  - **Compression Analysis**
- **The invertibility of embedding space**
  - **Real Problem in Digital Watermarking**
  - **Novel Watermarking Scheme**
- **The sensitivity of embedding space**
  - **Theoretical Deduction**
  - **Experimental results**

Thanks for your listening!