

# Functional Isolation Forest

Pavlo Mozharovskyi

joint with

Guillaume Staerman, Stephan Cléménçon, and Florence D'Alché-Buc

LTCI, Telecom Paris, Institut Polytechnique de Paris

Séminaire de statistique appliquée du CNAM

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- The task of anomaly detection

- Isolation forest

- AD in functional framework

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- FIF parameters

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## Extension of FIF

- Connection to data depth

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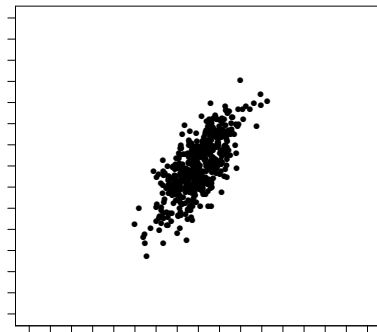
## Real data benchmarking

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# A real task

Regard two measurements during a test in a production process:

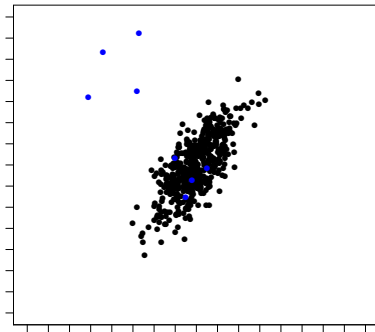


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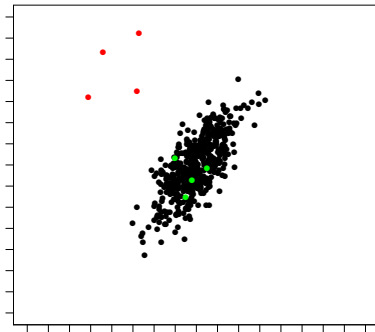
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For **new data**, determine:

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# Multivariate framework

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$$\mathbf{X} = \{\mathbf{x}_1, \dots, \mathbf{x}_n\} \subset \mathbb{R}^d$$

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$$\mathbb{R}^d \rightarrow \{-1, +1\} : \mathbf{x} \mapsto g(\mathbf{x}),$$

which attributes to any (possible)  $\mathbf{x} \in \mathbb{R}^d$  a label whether it is an anomaly (e.g., +1) or a normal observation (e.g., -1).

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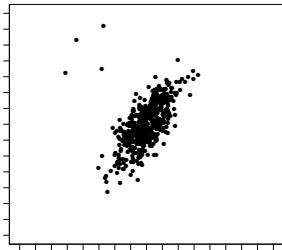
- ▶ It is more useful to provide an ordering on  $\mathbb{R}^d$ :

$$\mathbb{R}^d \rightarrow \mathbb{R} : \mathbf{x} \mapsto g(\mathbf{x}),$$

such that abnormal observations obtain higher anomaly score.

# Anomaly Detection (AD)

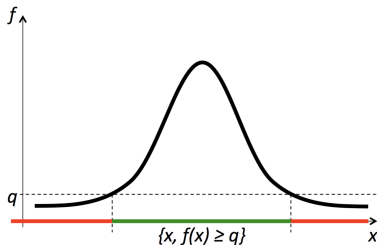
- ▶ What is **Anomaly detection**?
  - ▶ Identify unusual patterns that do not conform to expected behavior.
- ▶ Applications : Network intrusions, credit card fraud detection, insurance, finance, military surveillance, predictive maintenance, medical monitoring.



# Unsupervised Anomaly Detection

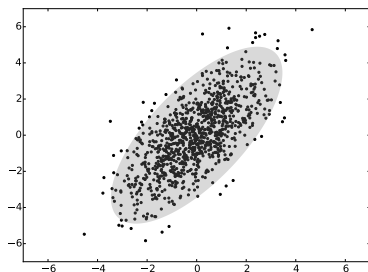
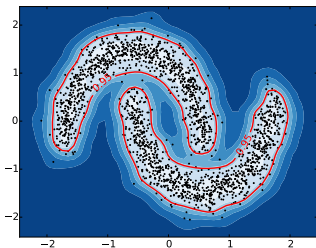
**Unsupervised Anomaly Detection:** data are **unlabelled**. We suppose that all data come from the same distribution  $\mu$  and that anomalies are very rare, *i.e.*, belongs to the low density regions.

If  $\mu$  admits a density  $f$  w.r.t. a measure  $\rho$ , the goal of anomaly detection can be formulated as the recovery of upper-level sets  $\{x : f(x) \geq q\}$ ,  $q \geq 0$ .



# Statistical inference

- ▶ **Plug-in method** : Estimation by  $\{x : \hat{f}(x) \geq q\}$ .
- ▶ **Direct methods** : Build a score function  $s : \mathcal{X} \rightarrow \mathbb{R}$  such that  $\{x : s(x) \geq q\}$  is close to  $\{x : f(x) \geq q\}$ .



# Unsupervised AD in practice

► How to do it in practice?

**Step 1.** Learn a **score function**  $s : \mathcal{X} \rightarrow \mathbb{R}$  which assigns a score to each data.

**Step 2.** Find the best **threshold** to construct a decision function which separates "normal" and "abnormal" data and then induces **two regions**.

**Step 3.** **Detect anomalies** among new observations.

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# Isolation forest (Liu, Ting, Zhou; 2008)

- ▶ **Isolation forest** (Liu, Ting, Zhou; 2008) is an anomaly detection method inherited from the famous **random forest** algorithm (Breiman, 2001).
- ▶ Since no supervised feedback is given, isolation forest is based on **purely random** (uniform) variable-based partitioning.



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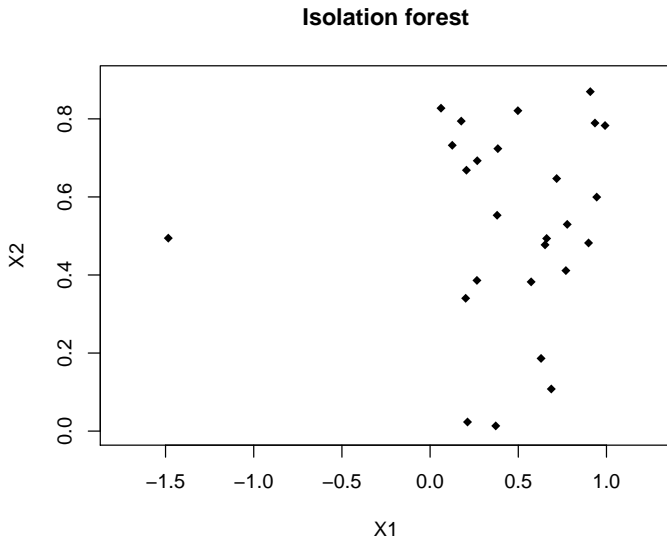
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- ▶ **Main idea:** **Outlying observations are isolated faster.**
- ▶ Tree-kind partitioning is done until “full isolation”: **outlying observations will have smaller depth** (on an average) in the **isolation tree**.
- ▶ A **monotone transform** is usually applied to the aggregated estimate.
- ▶ To reduce both **masking effect** and **computation cost**, small-size sub-sampling is used instead of bootstrap.

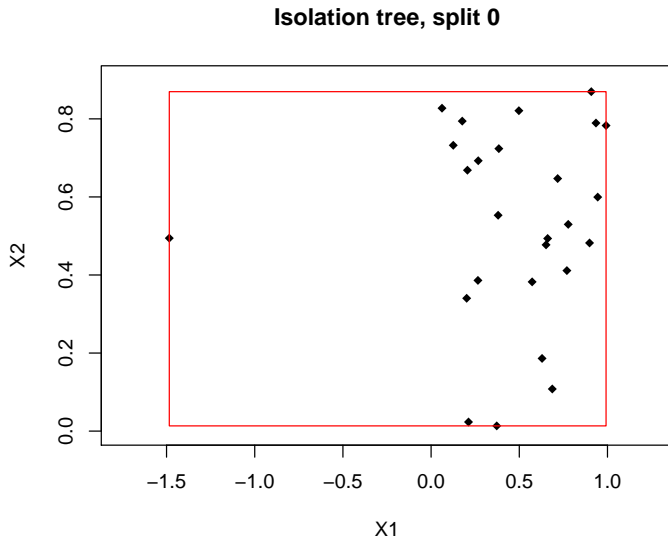
# Isolation forest (Liu, Ting, Zhou; 2008)

Illustration: Isolation tree



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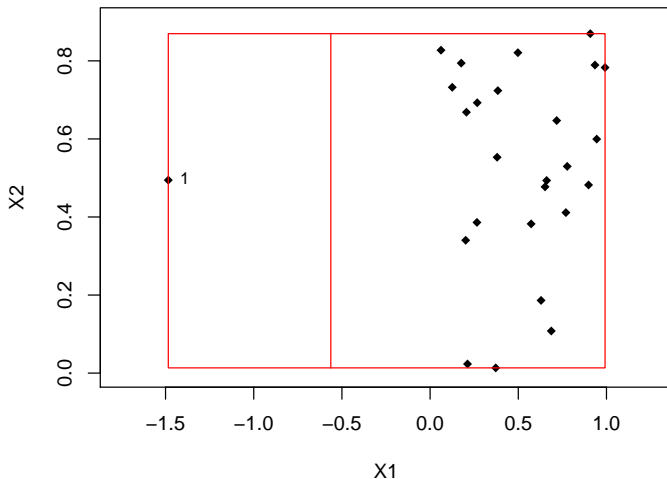
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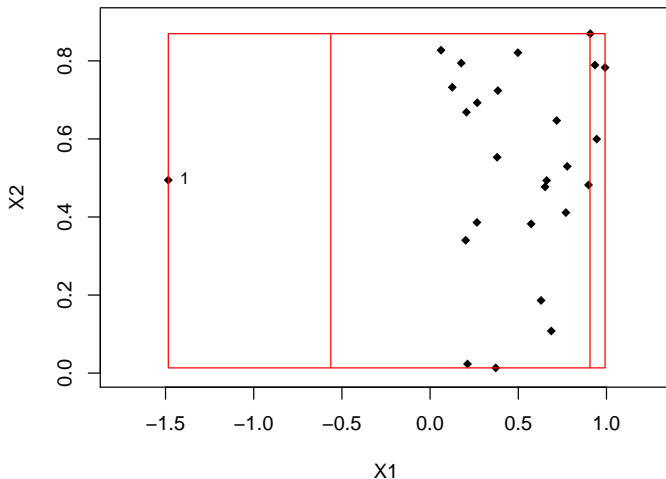
Isolation tree, split 1



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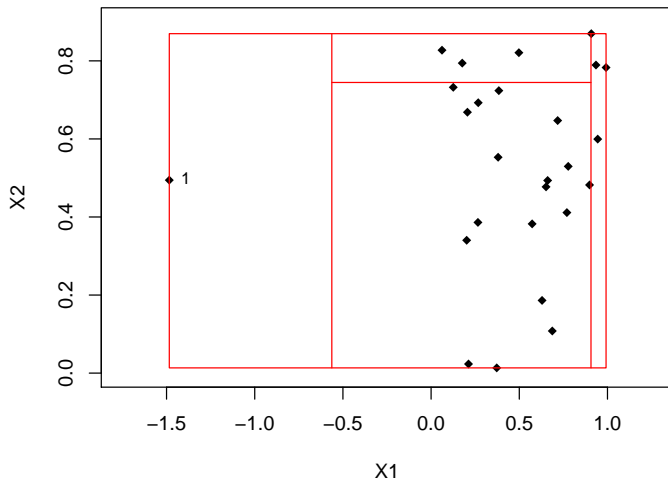
Isolation tree, split 2



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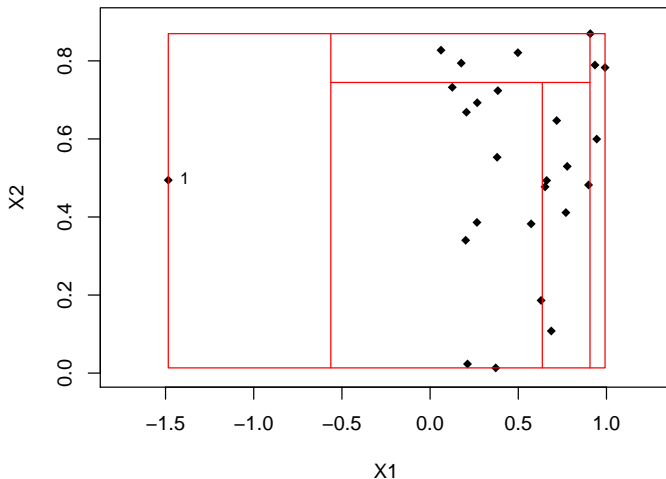
Isolation tree, split 3



# Isolation forest (Liu, Ting, Zhou; 2008)

## Illustration: Isolation tree

Isolation tree, split 4

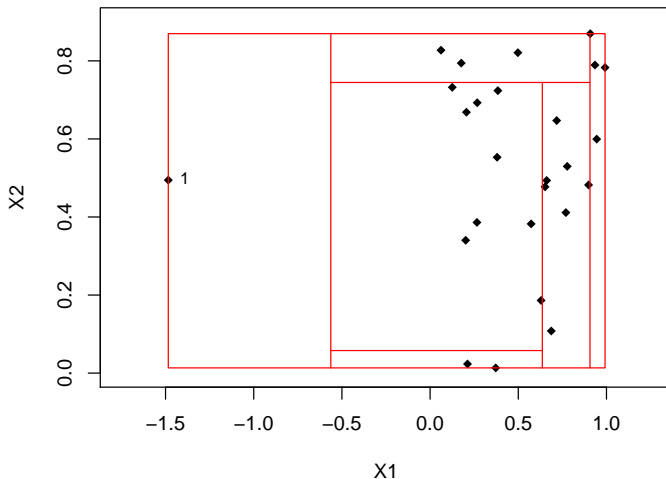




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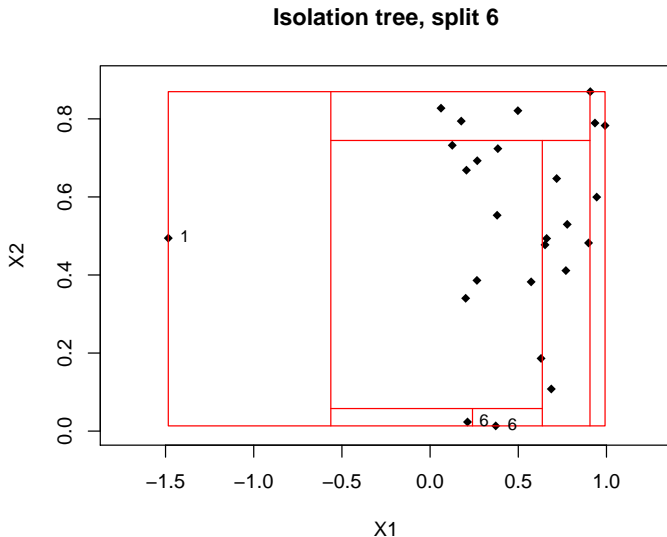
## Illustration: Isolation tree

Isolation tree, split 5



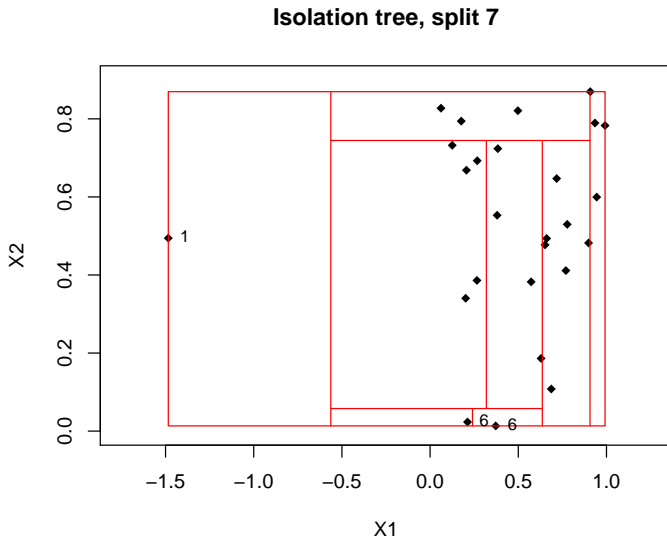
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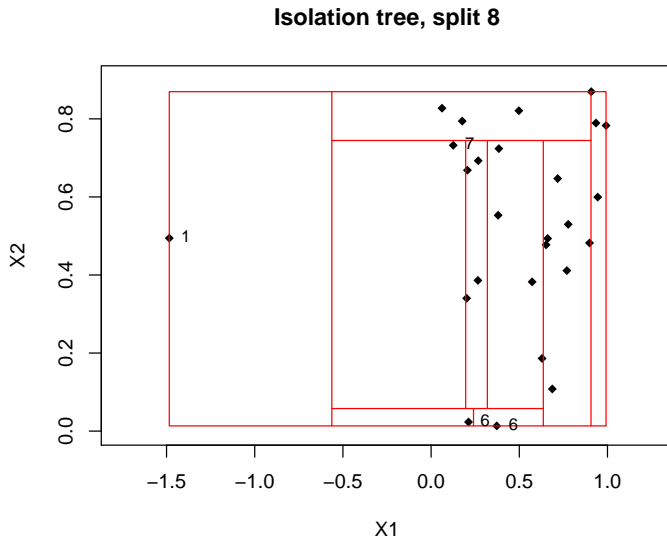
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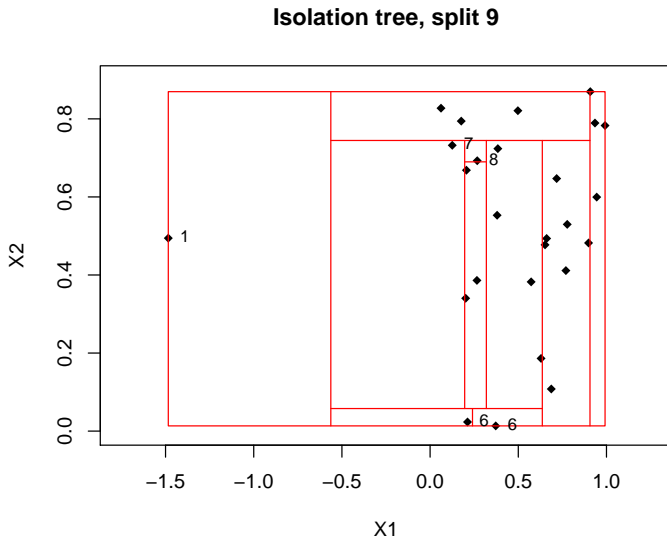
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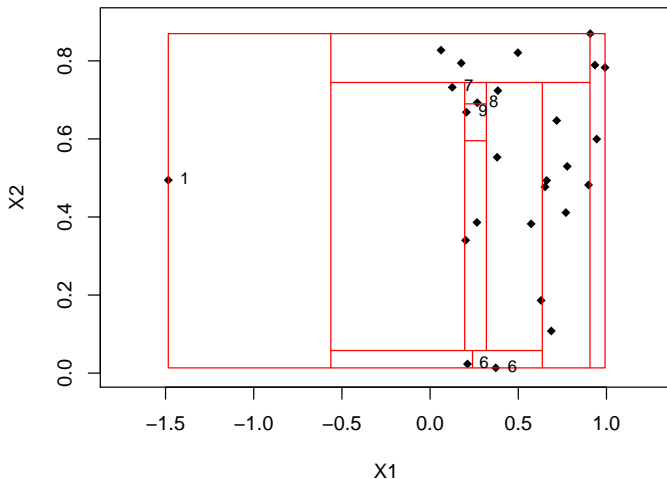
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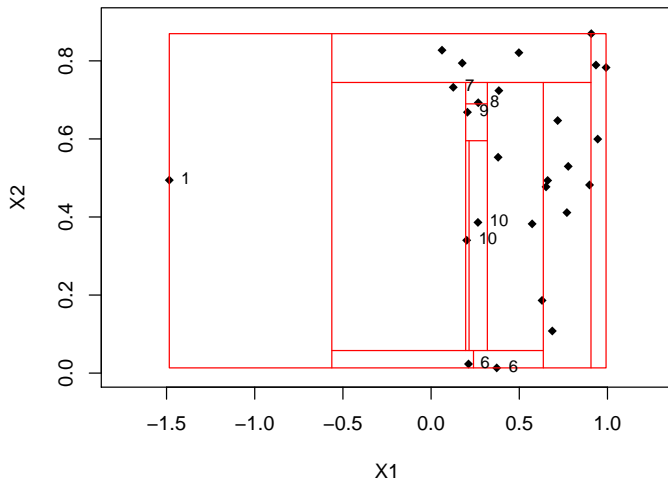
Isolation tree, split 10



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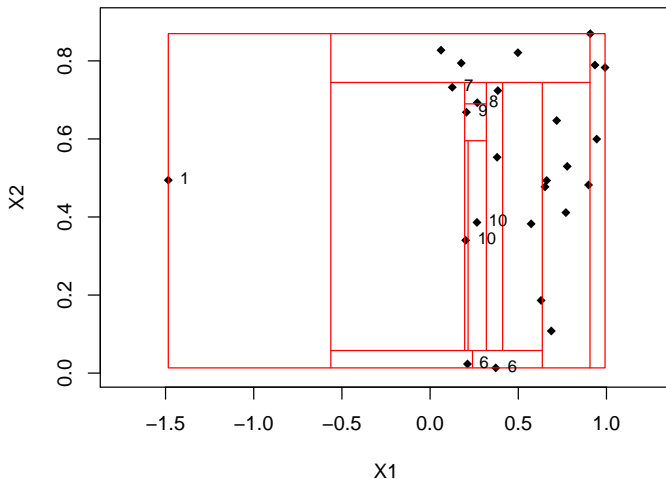
Isolation tree, split 11



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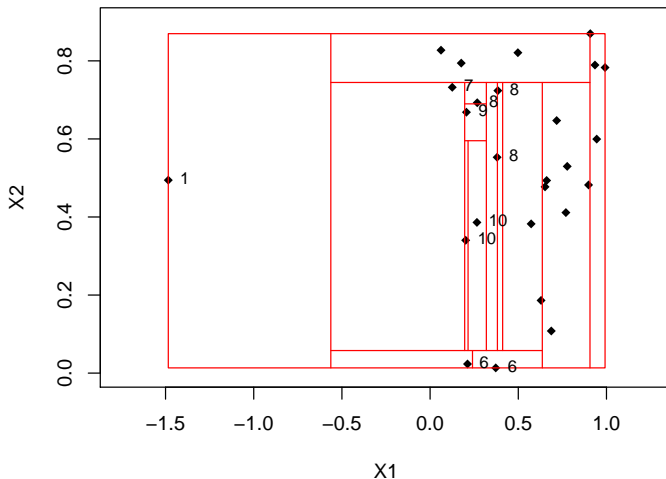
Isolation tree, split 12





### Illustration: Isolation tree

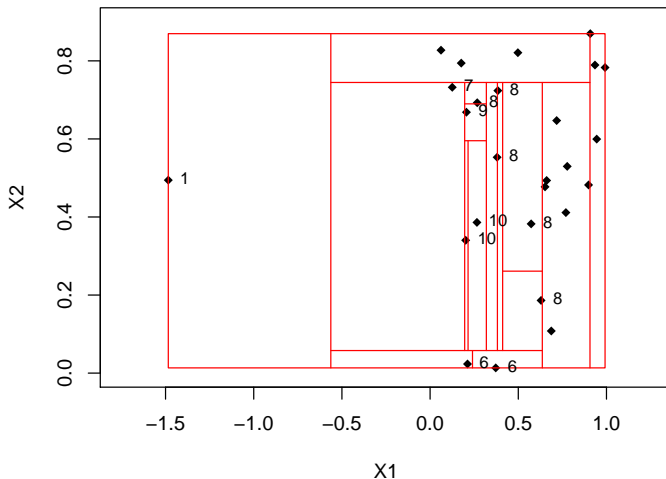
### Isolation tree, split 13



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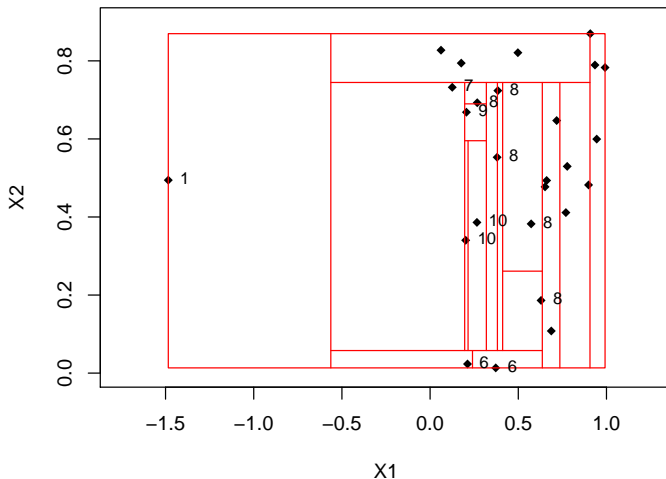
### Isolation tree, split 14



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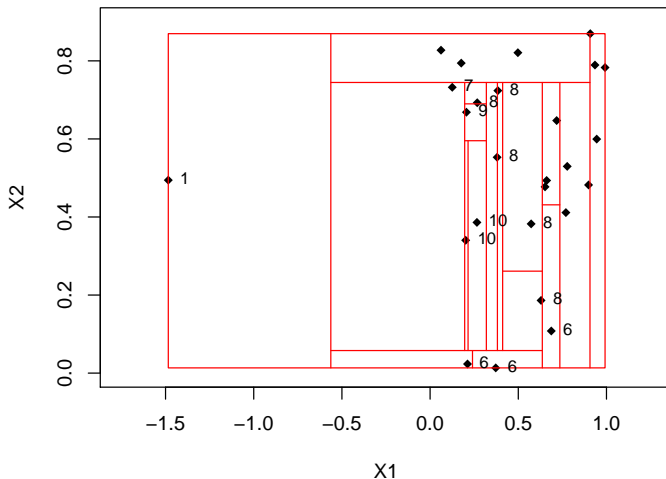
Isolation tree, split 15



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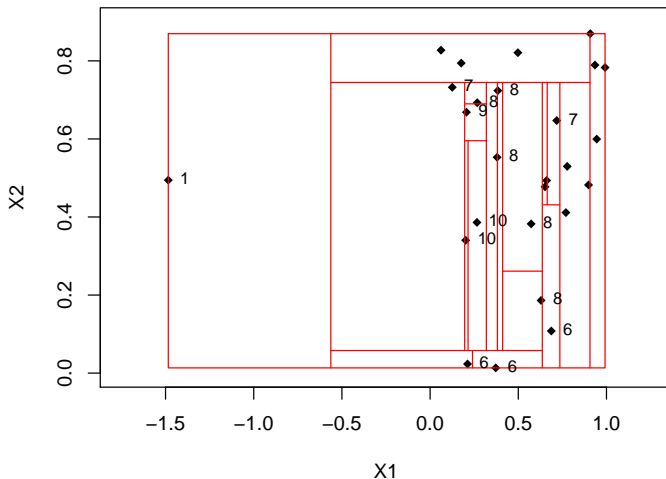
Isolation tree, split 16



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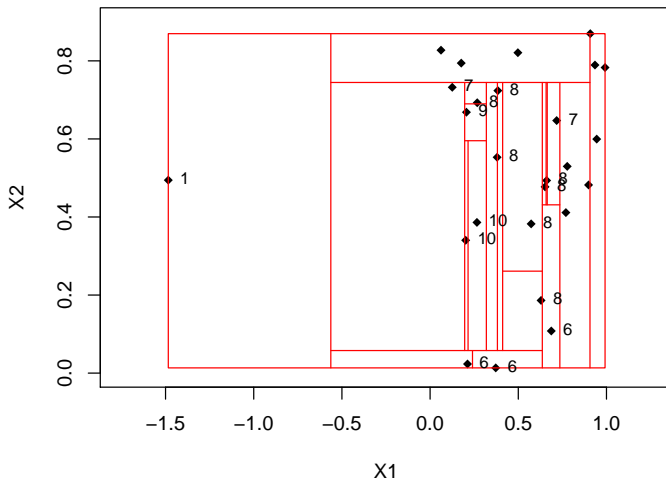
Isolation tree, split 17



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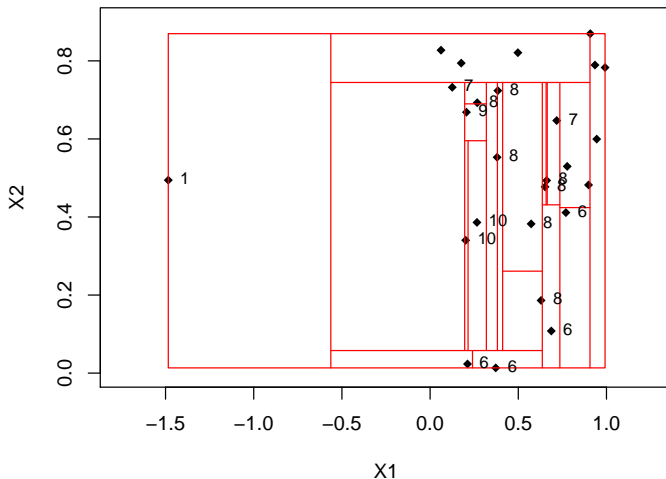
Isolation tree, split 18



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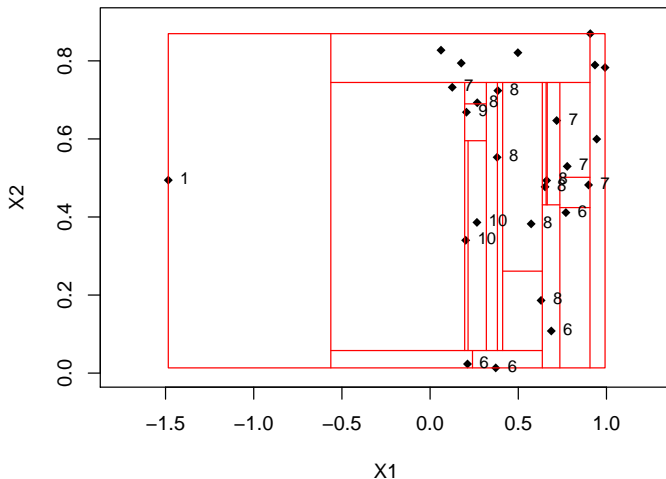
Isolation tree, split 19



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## Illustration: Isolation tree

Isolation tree, split 20

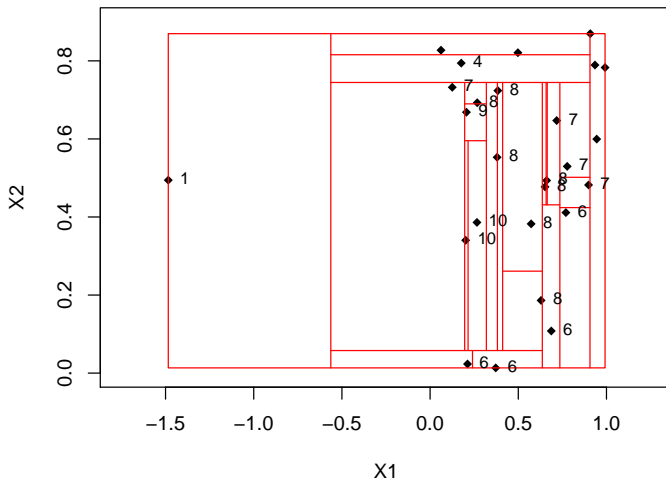




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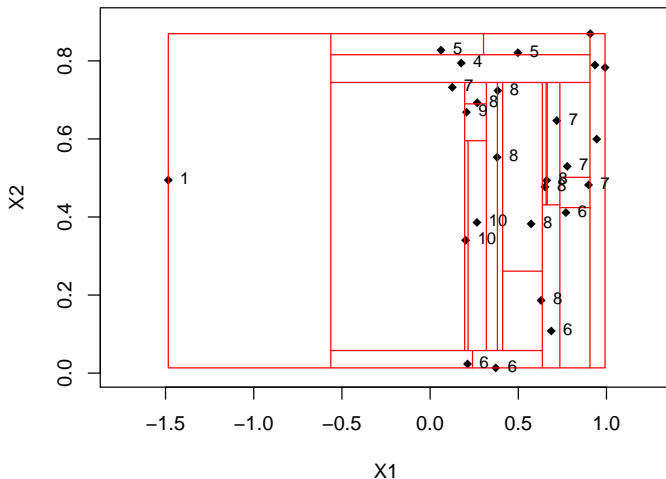
Isolation tree, split 21



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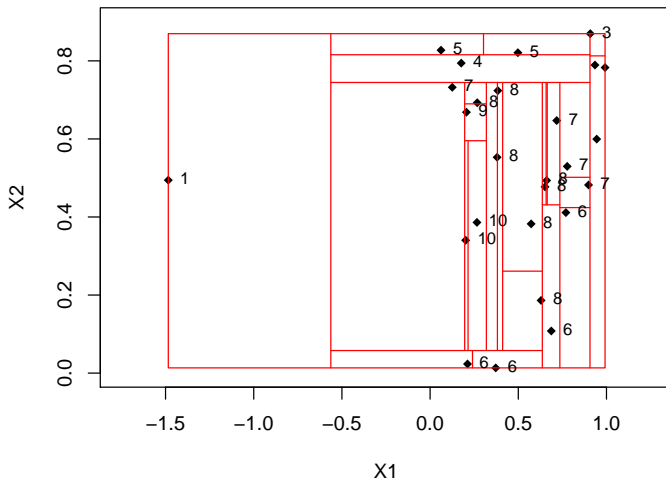
Isolation tree, split 22



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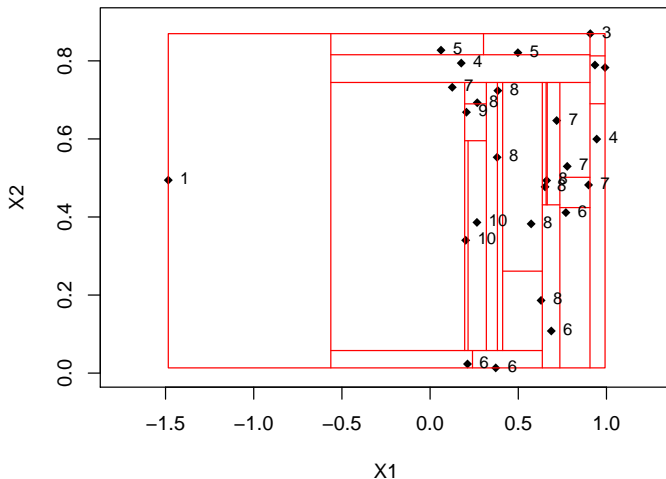
Isolation tree, split 23



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## Illustration: Isolation tree

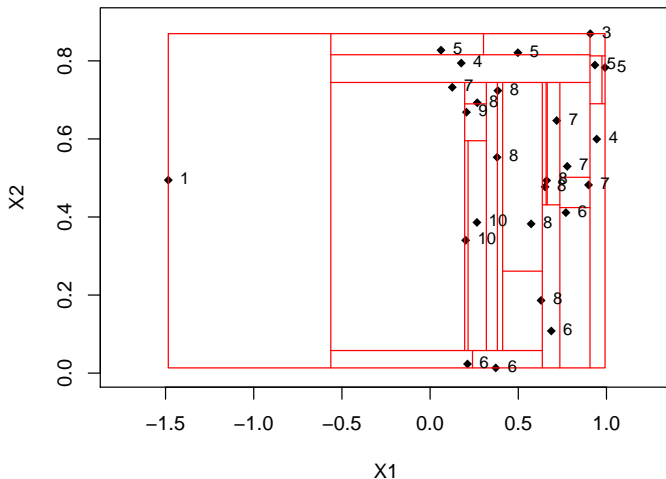
Isolation tree, split 24



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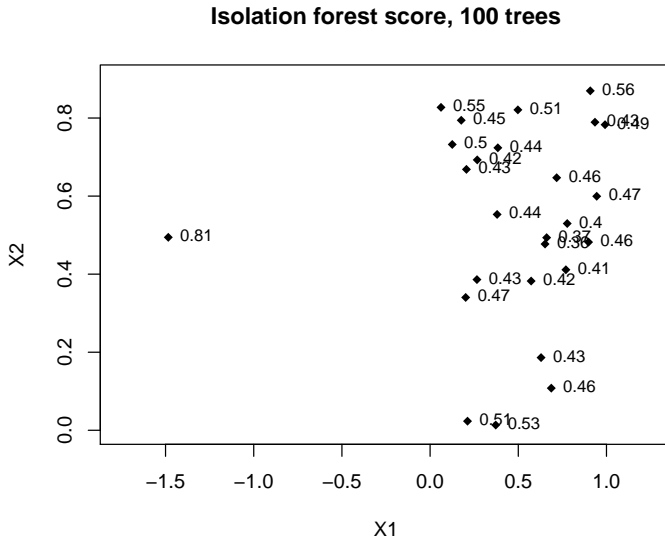
## Illustration: Isolation tree

Isolation tree, split 25



# Isolation forest (Liu, Ting, Zhou; 2008)

Illustration: Anomaly score



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# Functional Data Framework

- ▶ Let  $X$  be a functional random variable that takes its values in a functional space and  $X_1, \dots, X_n$  be an i.i.d. sample from  $X$ :

$$\begin{array}{lll} X & : & (\Omega, \mathcal{A}, \mathbb{P}) \longrightarrow \mathcal{H}([0, 1]) \\ & \omega & \longmapsto X(\omega) = (X_t(\omega))_{t \in [0, 1]}. \end{array}$$

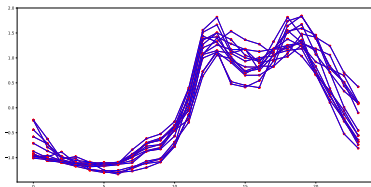


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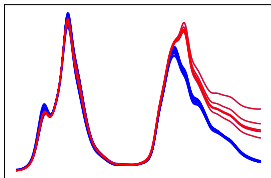
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- ▶ Data are basically **parametric curves**, *i.e.*, data collected in quasi-real time.
- ▶ In practice, we only have access to a finite number of arguments/times,  $\mathcal{S}_n = \{X_i(t_1), \dots, X_i(t_p), 1 \leq i \leq n\}$  such that  $0 \leq t_1 < \dots < t_p \leq 1$ .
- ▶ First step: reconstruct a functional object from time-series either by **interpolation** or **basis decomposition**.

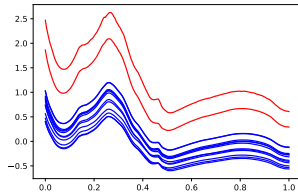


# Anomaly Detection and functional data (Hubert et al., 2015)

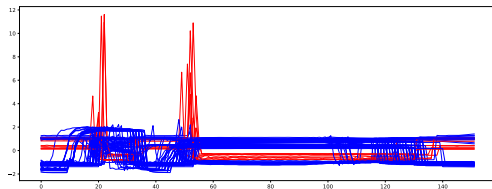
Shape anomalies



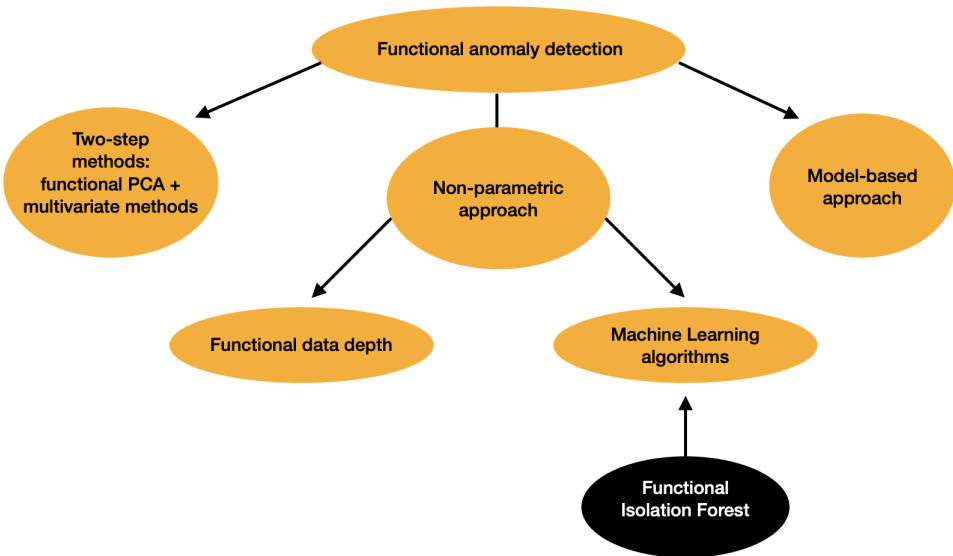
Shift anomalies



Isolated anomalies



# Context of FAD contributions



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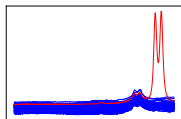
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- ▶  $X_1, \dots, X_n$  are random variables in Hilbert space  $\mathcal{H}$  and  $\mathcal{D} \subset \mathcal{H}$ .
- ▶ This **ensemble learning** algorithm builds a collection of binary tree based on a recursive and **randomized** tree-structured partitioning procedure.


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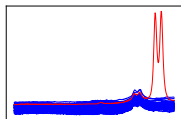
  $\mathbb{R}$

$\{(X_i, \mathbf{d})_{\mathcal{H}}, i \leq n\}$

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Step 2:

$$\begin{array}{c} \text{---} \times \times \times \times \text{---} \times \text{---} \mathbb{R} \\ \{ \langle X_i, \mathbf{d} \rangle_{\mathcal{H}}, i \leq n \} \end{array} \xrightarrow{\text{Draw uniformly } \gamma} \begin{array}{c} \gamma \\ \text{---} \times \times \times \times \text{---} \times \text{---} \mathbb{R} \\ \{x: \langle x, \mathbf{d} \rangle_{\mathcal{H}} \leq \gamma\} \quad \{x: \langle x, \mathbf{d} \rangle_{\mathcal{H}} > \gamma\} \end{array}$$

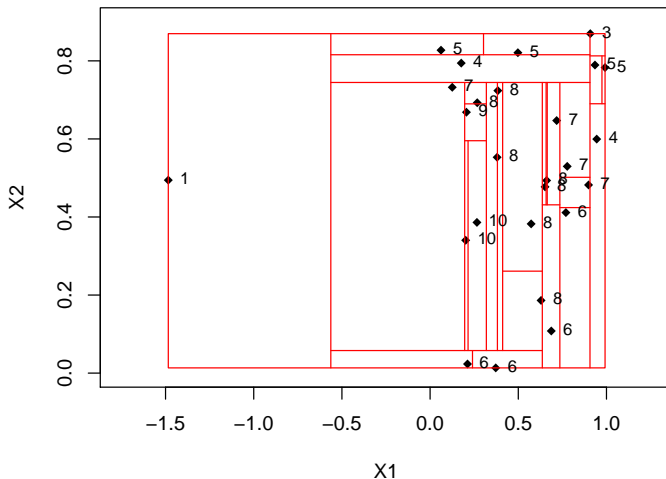
- ▶ The trick: an anomaly should be isolated faster than normal data.



# Functional Isolation Forest

## Illustration: Isolation tree

Isolation tree, split 25



# Children node construction in a functional isolation tree

If a node  $(j, k)$  is **non terminal**, it is split in three steps as follows:

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3. Form the children subsets

$$\begin{aligned} \mathcal{C}_{j+1,2k} &= \mathcal{C}_{j,k} \cap \{\mathbf{x} \in \mathcal{H} : \langle \mathbf{x}, \mathbf{d} \rangle_{\mathcal{H}} \leq \gamma\}, \\ \mathcal{C}_{j+1,2k+1} &= \mathcal{C}_{j,k} \cap \{\mathbf{x} \in \mathcal{H} : \langle \mathbf{x}, \mathbf{d} \rangle_{\mathcal{H}} > \gamma\}. \end{aligned}$$

as well as the children training datasets

$$\mathcal{S}_{j+1,2k} = \mathcal{S}_{j,k} \cap \mathcal{C}_{j+1,2k} \text{ and } \mathcal{S}_{j+1,2k+1} = \mathcal{S}_{j,k} \cap \mathcal{C}_{j+1,2k+1}.$$

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If a node  $(j, k)$  is **non terminal**, it is split in three steps as follows:

1. Choose a **Split function**  $\mathbf{d}$  according to the probability distribution  $\nu$  on  $\mathcal{D}$ .
2. Choose randomly and uniformly a **Split value**  $\gamma$  in the interval

$$\left[ \min_{\mathbf{x} \in \mathcal{S}_{j,k}} \langle \mathbf{x}, \mathbf{d} \rangle_{\mathcal{H}}, \max_{\mathbf{x} \in \mathcal{S}_{j,k}} \langle \mathbf{x}, \mathbf{d} \rangle_{\mathcal{H}} \right],$$

3. Form the children subsets

$$\begin{aligned} \mathcal{C}_{j+1,2k} &= \mathcal{C}_{j,k} \cap \{\mathbf{x} \in \mathcal{H} : \langle \mathbf{x}, \mathbf{d} \rangle_{\mathcal{H}} \leq \gamma\}, \\ \mathcal{C}_{j+1,2k+1} &= \mathcal{C}_{j,k} \cap \{\mathbf{x} \in \mathcal{H} : \langle \mathbf{x}, \mathbf{d} \rangle_{\mathcal{H}} > \gamma\}. \end{aligned}$$

as well as the children training datasets

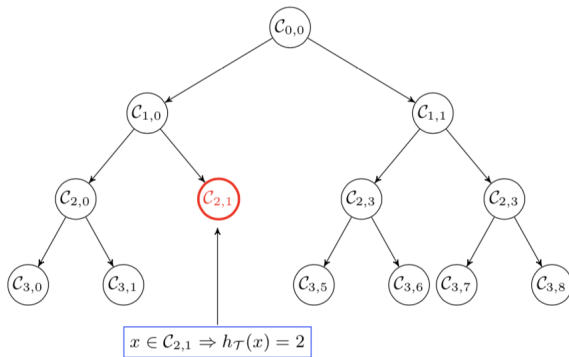
$$\mathcal{S}_{j+1,2k} = \mathcal{S}_{j,k} \cap \mathcal{C}_{j+1,2k} \text{ and } \mathcal{S}_{j+1,2k+1} = \mathcal{S}_{j,k} \cap \mathcal{C}_{j+1,2k+1}.$$

**Stop** when only one observation is in each node: **isolation**.

# Anomaly score prediction

- One may then define the **piecewise constant function**  $h_{\tau} : \mathcal{H} \rightarrow \mathbb{N}$  by:  
 $\forall \mathbf{x} \in \mathcal{H}$ ,

$h_{\tau}(\mathbf{x}) = j$  if and only if  $\mathbf{x} \in \mathcal{C}_{j,k}$  and  $\mathcal{C}_{j,k}$  is associated to a terminal node.



# Anomaly score prediction

**Anomaly score calculation** for observation  $\mathbf{x}$ :

1. For each **isolation tree**  $i \in \{1, \dots, N\}$ , locate  $\mathbf{x}$  in a **terminal node** and calculate the **depth** of this node  $h_i(\mathbf{x})$ .
2. Attribute the **anomaly score**:

$$s_n(\mathbf{x}) = 2^{-\frac{1}{N \cdot c(n)} \sum_{i=1}^N h_i(\mathbf{x})},$$

with  $c(n) = 2H(n-1) - \frac{2(n-1)}{n}$  where  $H(k)$  is the harmonic number and can be estimated by  $\ln(k) + 0.5772156649$ .

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**Score behavior:**

- ▶ when  $\frac{1}{N} \sum_{i=1}^N h_i(\mathbf{x}) \rightarrow c(n)$ ,  $s_n(\mathbf{x}) \rightarrow 0.5$ ,
- ▶ when  $\frac{1}{N} \sum_{i=1}^N h_i(\mathbf{x}) \rightarrow 0$ ,  $s_n(\mathbf{x}) \rightarrow 1$ ,
- ▶ when  $\frac{1}{N} \sum_{i=1}^N h_i(\mathbf{x}) \rightarrow n-1$ ,  $s_n(\mathbf{x}) \rightarrow 0$ .



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# Parameters of FIF

- ▶ Classical parameters of ISOLATION FOREST :
  - ▶ The number of trees, the size of the subsample and the height limit.
- ▶ New parameters due to the functional setup :
  1. The **dictionary**  $\mathcal{D}$ .
  2. The **probability measure**  $\nu$ .
  3. The **scalar product**  $\langle \cdot, \cdot \rangle_{\mathcal{H}}$ .

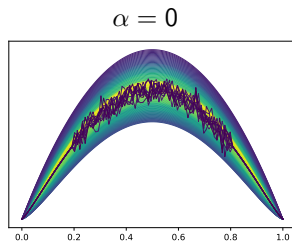
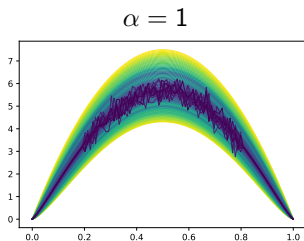
# The role of the scalar product

- Compromise between both location and shape :

$$\langle \mathbf{f}, \mathbf{g} \rangle := \alpha \times \frac{\langle \mathbf{f}, \mathbf{g} \rangle_{L_2}}{\|\mathbf{f}\| \|\mathbf{g}\|} + (1 - \alpha) \times \frac{\langle \mathbf{f}', \mathbf{g}' \rangle_{L_2}}{\|\mathbf{f}'\| \|\mathbf{g}'\|}, \quad \alpha \in [0, 1],$$

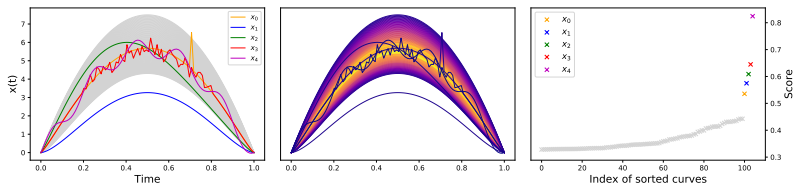
## Example on a toy dataset :

- 90 curves defined by  $\mathbf{x}(t) = 30(1 - t)^q t^q$  with  $q$  equispaced in  $[1, 1.4]$ ,
- 10 *abnormal* curves defined by  $\mathbf{x}(t) = 30(1 - t)^{1.2} t^{1.2}$  noised by  $\varepsilon \sim \mathcal{N}(0, 0.3^2)$  on the interval  $[0.2, 0.8]$ .



# Ability to detect a variety of anomalies

- ▶ Sobolev inner product:  $\langle \cdot, \cdot \rangle_{W_{1,2}}$ .
- ▶ Gaussian wavelets dictionary  $d_{\theta, \sigma}(t) = \frac{2}{\sqrt{3}\sigma\pi^{1/4}} \left(1 - \left(\frac{t-\theta}{\sigma}\right)^2\right) \exp\left(\frac{-(t-\theta)^2}{2\sigma^2}\right)$ .
- ▶ Uniform measure  $\nu$ .



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# Performance on real datasets (1)

- ▶ **FIF** with 4 setups (Dictionary+scalar product):
  - ▶ Dyadic indicator (**DI**)+ $L_2$
  - ▶ Cosine (**Cos**)+ $L_2$
  - ▶ Cosine (**Cos**)+Sobolev
  - ▶ Dataset itself (**Self**)+ $L_2$

## Competitors:

- ▶ *Isolation Forest*, *Local Outlier Factor*, *One-class SVM*  
after dimension reduction by FPCA.
- ▶  $fHD_{RP}$  : Random projection method with functional Halspace depth.
- ▶ **fSDO** : Functional Stahel-Donoho Outlyingness.

## Performance on real datasets (2)

Methods :	$DI_{L_2}$	$Cos_{Sob}$	$Cos_{L_2}$	$Self_{L_2}$	IF	LOF	OCSVM	$fHD_{RP}$	fSDO
Chinatown	0.93	0.82	0.74	0.77	0.69	0.68	0.70	0.76	0.98
Coffee	0.76	0.87	0.73	0.77	0.60	0.51	0.59	0.74	0.67
ECGFiveDays	0.78	0.75	0.81	0.56	0.81	0.89	0.90	0.60	0.81
ECG200	0.86	0.88	0.88	0.87	0.80	0.80	0.79	0.85	0.86
Handoutlines	0.73	0.76	0.73	0.72	0.68	0.61	0.71	0.73	0.76
SonyRobotAI1	0.89	0.80	0.85	0.83	0.79	0.69	0.74	0.83	0.94
SonyRobotAI2	0.77	0.75	0.79	0.92	0.86	0.78	0.80	0.86	0.81
StarLightCurves	0.82	0.81	0.76	0.86	0.76	0.72	0.77	0.77	0.85
TwoLeadECG	0.71	0.61	0.61	0.56	0.71	0.63	0.71	0.65	0.69
Yoga	0.62	0.54	0.60	0.58	0.57	0.52	0.59	0.55	0.55
EOGHorizontal	0.72	0.76	0.81	0.74	0.70	0.69	0.74	0.73	0.75
CinECGTorso	0.70	0.92	0.86	0.43	0.51	0.46	0.41	0.64	0.80
ECG5000	0.93	0.98	0.98	0.95	0.96	0.93	0.95	0.91	0.93

**Table:** AUC of different anomaly detection methods calculated on the test set.  
Bold numbers correspond to the best result.

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# Extension to multivariate functional data

FIF can be easily extended to the multivariate functional data, *i.e.* when the quantity of interest lies in  $\mathbb{R}^d$  for each moment of time:

$$\begin{aligned}x &: [0, 1] \longrightarrow \mathbb{R}^d \\t &\longmapsto ((x^1(t), \dots, x^d(t)))\end{aligned}$$

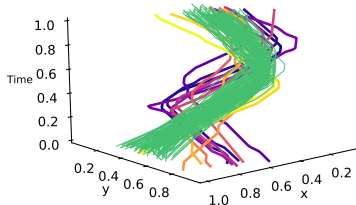
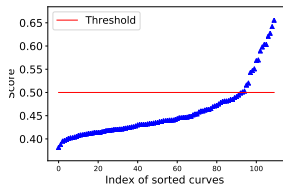
- Coordinate-wise sum of the  $d$  corresponding scalar products:

$$\langle \mathbf{f}, \mathbf{g} \rangle_{L_2^{\otimes d}} := \sum_{i=1}^d \langle f^{(i)}, g^{(i)} \rangle_{L_2}$$

- Dictionaries : Composed by univariate function on each axis, multivariate wavelets, multivariate Brownian motion ...

# Example with MNIST dataset

We extract the digits' contours and obtain bivariate functional curves from MNIST dataset. Each digit is transformed into a curve in  $(L_2([0, 1]) \times L_2([0, 1]))$  using length parametrization on  $[0, 1]$ .



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# Connection to data depth and supervised classification

- ▶ One may define a **functional depth** by  $D_{FIF}(x; \mathcal{S}) = 1 - s_n(x; \mathcal{S})$ .

Assume that we have a training classification dataset of  $q$  classes  
 $\mathcal{S} = \mathcal{S}^1 \cup \dots \cup \mathcal{S}^q$ .

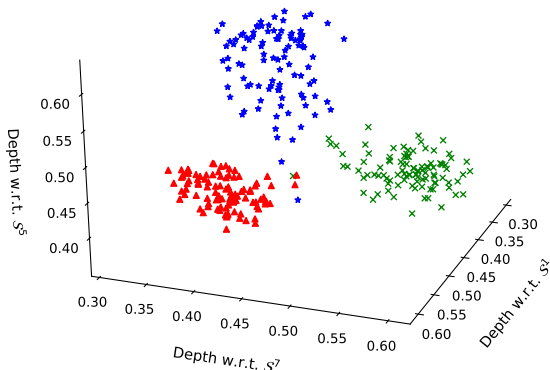
- ▶ Low dimensional representation based on **depth-based map** can be defined by

$$\mathbf{x} \mapsto \phi(\mathbf{x}) = (D_{FIF}(\mathbf{x}; \mathcal{S}^1), \dots, D_{FIF}(\mathbf{x}; \mathcal{S}^q)) \in [0, 1]^q.$$

- ▶ One may define a **DD-plot classifier** by using a classifier on the low dimension representation of the functional dataset.

## Example of depth map on MNIST dataset

$\mathcal{S}$  is constructed by taking 100 digits from class 1, 100 from class 5 and 100 from class 7.



**Figure:** Depth space embedding of the three digits (1, 5 and 7) of the MNIST dataset.

# Conclusion

- ▶ New anomaly detection algorithm for functional data:
  - ▶ Great **flexibility** but dictionaries (and scalar product) are tricky to choose in an unsupervised setting.
  - ▶ Low **complexity** and **memory requirement**.
- ▶ Lack of theoretical guarantees!

STAERMAN, G., MOZHAROVSKIY, P., CLÉMENÇON, S., AND D'ALCHÉ-BUC, F. **Functional Isolation Forest**. *ACML 2019*.

All codes are available at:

<https://github.com/guillaumestaermanML/FIF>.

Thank you for your attention!

Questions?