Conjoint Analysis with auxiliary information

- The scheme of analysis we propose is basically composed by three main matrices:

① The matrix **Y** (S, J) holds the overall evaluation of the J judges about S stimuli profiles.

② The matrix of predictor variables, **X** (S, K). In the Design of Experiment framework, **X** is a Design Matrix, and it retrieves information coded according to suitable indicators (*dummy*) variables.

Conjoint Analysis with auxiliary information

③ The third dataset is constituted by auxiliary information collected on respondents to the Conjoint task. These information are related to each column of Y, so they should considered as row-vector variables.

Giordano & Scepi (1999) proposed a factorial approach for taking into account simultaneously the information on judges and the characteristics of products in the definition of preferences.

Here we start from this approach for introducing a new scheme based on Relational data

The Network-Conjoint Model

$$\mathbf{B} = \Delta_X^{-1}\mathbf{X}'\mathbf{Y}$$

$$\mathbf{B}_{(N\times K)} = \mathbf{A}_{(N\times N)}\Theta_{(N\times K)} + err$$

$$\Theta = \Delta_{\mathrm{A}}^{-1}\mathbf{A}'\mathbf{B} \equiv \Delta_{\mathrm{A}}^{-1}\mathbf{A}'\Delta_X^{-1}\mathbf{X}'\mathbf{Y} \qquad \Delta_{\mathrm{A}}^{-1} = [Diag(\mathbf{A}'\mathbf{A})]^{-1}$$

A

	1	2	i	j	...	N
1	0					
2		0				
i			a_{ii}	a_{ij}		
j			a_{ji}	0		
...					...	
N						0

Θ

X_{11}	X_{12}	X_{21}	X_{22}	X_{23}	...	X_{F1}	X_{F2}

The elements of $\mathbf{\Theta}$ are the average of the part-worth coefficients for the neighbors of the generic judge

Y

Y_1	Y_2	...	Y_N
y_{11}	y_{12}	...	y_{1N}
y_{21}	y_{22}	...	y_{2N}
y_{31}	y_{32}	...	y_{3N}
y_{41}	y_{42}	...	y_{4N}
...	...	...	...
y_{S1}	y_{S2}	...	y_{SN}

X

X_{11}	X_{12}	X_{21}	X_{22}	X_{23}	...	X_{K1}	X_{K2}
1	0	1	0	0	...	1	0
1	0	0	1	0	...	0	1
1	0	0	0	1	...	1	0
0	1	1	0	0	...	0	1
...	...	...	...	...	...	...	...
0	1	0	0	1	...	0	1

The Adjacency matrix as auxiliary information in CA

If we want to understand variation in the preference of consumers, we need to take a closer look at their local circumstances. Describing and indexing the variation across individuals in the way they are embedded in "local" social structures is the goal of the **analysis of ego networks.**

Our aim is to consider the dyadic information derived from the Adjacency Matrix **A** as relational constraints on a set of individuals looked at both as Consumers and Social Actors in a Community.

We wish to explore whether and how the relational ties (information sharing, etc.) are somewhat related to the preference elicitation process.

Ego-network

"Ego" is an individual "focal" node. A network has as many egos as it has nodes. Egos can be persons, groups, organizations, or whole societies.

"Neighborhood" is the collection of ego and all nodes to whom ego has a connection at some path length. In social network analysis, the "neighborhood" is almost always one-step; that is, it includes only ego and actors that are directly adjacent.

The neighborhood also includes all of the ties among all of the actors to whom ego has a direct connection.

The Strategy

Obtain the estimates part-worth coefficients of the individual Conjoint Analysis Models

For each Consumer in the Net:

- Build the Ego-Net
- Aggregate the Conjoint Models for the neighbouring of ego
- Build the Ego-net prototype model
- Compare the individual model with the ego-net model

Network-Conjoint models

- The neighbour-effect solution:

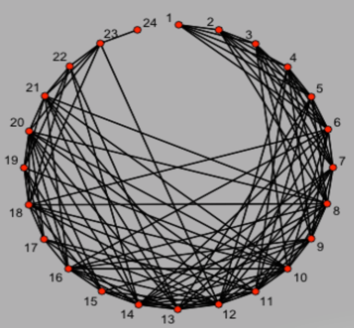
$$\mathbf{B}'_{(N\times K)} = \mathbf{A}_{(N\times N)}\Theta_{(N\times K)} + err$$

$$\Theta = \Delta_{\mathrm{A}}^{-1}\mathbf{AB}' \equiv \Delta_{\mathrm{A}}^{-1}\mathbf{AY}'\mathbf{X}\Delta_{X}^{-1} \qquad \Delta_{\mathrm{A}}^{-1} = [Diag(\mathbf{A}'\mathbf{A})]^{-1}$$

Is used to compare the Ego-Models with the Neighbors-Models according to the Local Differences: $\mathbf{B}' - \Theta = \mathbf{B}' - \Delta_{\mathrm{A}}^{-1}\mathbf{AB}'$

The deviation of the ego-model from the correspondent neighbors average model allows to explore the residual relevant attribute levels for the judge once the network effect has been eliminated

Toy-Example



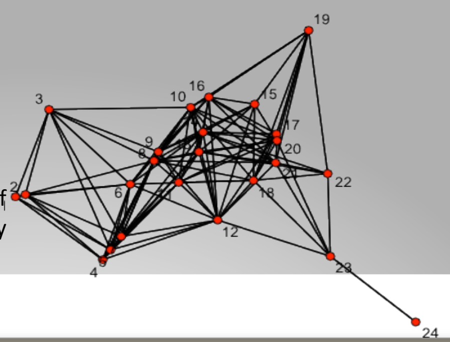
24 Nodes

Example: consumers embedded in a Social Network exchanging information about product's characteristics

167 Links:

Posts on a Specialised Forum

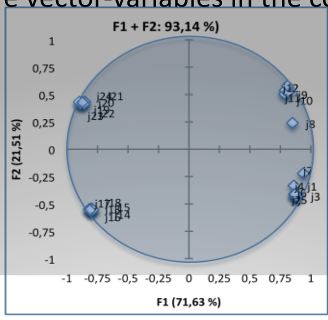
We simulate 4 different groups of consumers with respect to their utility models



Individual Conjoint Models

B	j1	j2	j3	j4	j5	j6	j7	j8	j9	j10	j11	j12	j13	j14	j15	j16	j17	j18	j19	j20	j21	j22	j23	j24
x11	-2,67	-3,00	-3,00	-3,00	-3,00	-3,00	-2,33	-1,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	3,00	3,00	3,00	3,00	3,00	3,00
x12	-0,33	0,00	0,00	0,00	0,00	0,00	-0,67	-2,00	-3,00	-3,00	-2,67	-3,33	3,00	3,00	3,00	3,00	3,00	3,00	0,00	0,00	0,00	0,00	0,00	0,00
x13	3,00	3,00	3,00	3,00	3,00	3,00	3,00	3,00	3,00	3,00	3,00	3,00	-3,00	-3,00	-3,00	-3,00	-3,00	-3,00	-3,00	-3,00	-3,00	-3,00	-3,00	-3,00
x21	-1,00	-0,67	-0,33	-0,33	0,00	0,67	0,00	1,00	0,00	0,00	-0,67	0,00	-0,67	-0,67	0,67	0,00	0,33	0,33	0,00	-0,33	0,67	0,00	-0,67	-0,33
x22	0,00	0,00	0,33	0,33	-0,33	0,00	0,33	-0,33	-1,00	0,33	1,00	-0,33	0,67	0,00	-1,00	0,33	-1,00	-0,33	-0,33	1,00	0,00	0,00	0,33	0,67
x23	1,00	0,67	0,00	0,00	0,33	-0,67	-0,33	-0,67	1,00	-0,33	0,00	0,00	0,00	0,67	0,33	-0,33	0,67	0,00	0,33	-0,67	-0,67	0,00	0,33	-0,33
x31	0,00	0,00	-0,33	1,00	0,00	0,33	0,33	-0,33	0,00	1,00	0,00	0,00	0,33	0,00	0,33	0,33	-0,33	0,00	0,00	0,00	0,33	0,00	0,33	-0,33
x32	0,67	-0,67	0,00	-0,67	-0,67	0,00	0,33	-1,00	0,00	-0,33	0,00	-0,33	0,00	0,33	0,00	0,00	0,00	-0,33	0,33	-0,33	0,00	0,00	-0,67	0,67
x33	-0,67	0,67	0,33	-0,33	0,67	-0,33	-0,67	1,33	0,00	-0,67	0,33	0,00	-0,33	-0,33	-0,33	-0,33	0,33	0,33	-0,33	0,33	-0,33	0,00	0,33	-0,33

PCA representation of matrix B:
Judgements are vector-variables in the correlation circle



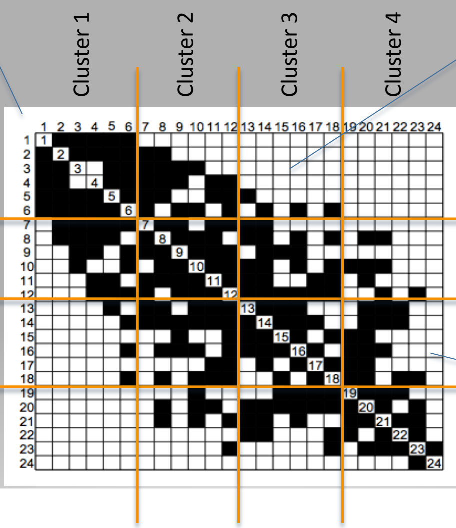
The four groups are all strongly separated

Inter and Intra-classes cohesion

High Intra–Class Cohesion

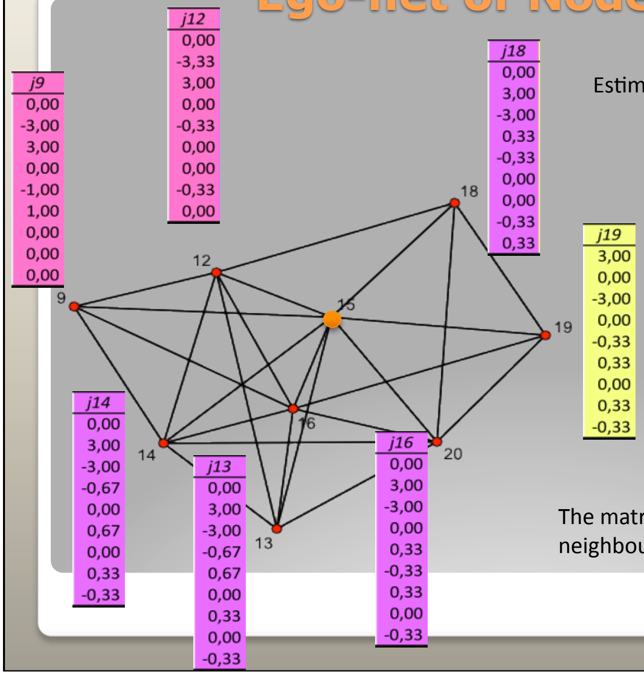
Low Inter–Classes Cohesion

Cluster 1
Cluster 2
Cluster 3
Cluster 4



High Inter–Classes Cohesion

Ego-net of Node 15



Estimated Coefficients of the Ego-Net J15

j15
0,00
3,00
-3,00
0,67
-1,00
0,33
0,33
0,00
-0,33

The matrix Θ holds the average of the 8 neighbours' coefficients

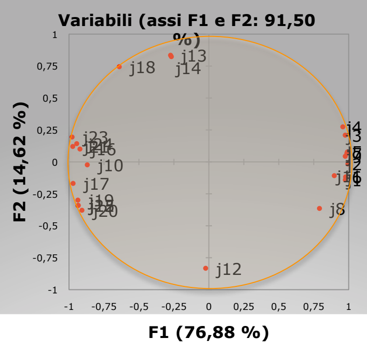
PCA of Neighbour Models

Ego	j1	j2	j3	j4	j5	j6	j7	j8	j9	j10	j11	j12
x11	-3,00	-2,57	-1,88	-1,67	-1,80	-1,42	-1,45	-0,60	-1,12	0,15	-0,53	-0,36
x12	0,00	-0,43	-1,13	-1,33	-0,60	-0,11	-0,45	0,19	-0,24	0,08	0,00	0,58
x13	3,00	3,00	3,00	3,00	2,40	1,50	1,91	0,43	1,36	-0,23	0,50	-0,20
x21	-0,13	0,00	0,00	-0,07	-0,20	-0,22	-0,15	-0,19	0,03	0,00	-0,03	-0,02
x22	0,07	0,05	-0,13	-0,07	0,20	-0,03	0,03	0,19	0,09	0,08	-0,06	-0,02
x23	0,07	-0,05	0,13	0,15	0,00	0,22	0,12	0,02	-0,12	-0,08	0,06	0,07
x31	0,20	0,14	0,17	0,04	0,13	0,19	0,09	0,21	0,24	0,05	0,17	0,27
x32	-0,40	-0,19	-0,21	-0,19	-0,17	-0,14	-0,27	-0,17	-0,15	-0,10	-0,25	-0,13
x33	0,20	0,05	0,04	0,15	0,03	-0,08	0,18	-0,02	-0,09	0,05	0,06	-0,11

Ego	j13	j14	j15	j16	j17	j18	j19	j20	j21	j22	j23	j24
x11	0,22	0,19	0,75	0,42	0,86	0,73	1,13	0,67	1,10	1,50	2,00	3,00
x12	-0,22	-0,19	0,71	0,06	0,43	-0,45	1,13	0,86	0,07	1,50	-0,06	0,00
x13	0,00	0,00	-1,50	-0,50	-1,29	-0,27	-2,25	-1,50	-1,20	-3,00	-2,00	-3,00
x21	0,06	0,12	-0,17	0,14	-0,19	0,09	0,21	0,03	-0,13	-0,17	0,06	-0,67
x22	0,08	0,00	0,00	-0,17	0,19	-0,03	-0,08	-0,03	0,17	0,06	0,17	0,33
x23	-0,14	-0,12	0,13	0,00	0,00	-0,06	-0,13	0,03	-0,07	0,11	-0,28	0,33
x31	0,17	0,17	0,08	0,14	0,05	0,12	0,21	0,17	0,17	0,17	0,00	0,33
x32	-0,17	-0,12	-0,04	-0,11	-0,05	-0,21	-0,13	-0,14	-0,20	-0,06	-0,06	-0,67
x33	0,00	-0,05	-0,08	-0,06	0,00	0,09	-0,08	0,00	0,00	-0,11	0,00	0,33

Each point represents the average preference model of the neighbours

Close Judges are equivalent in the sense that their neighbours have similar preferences



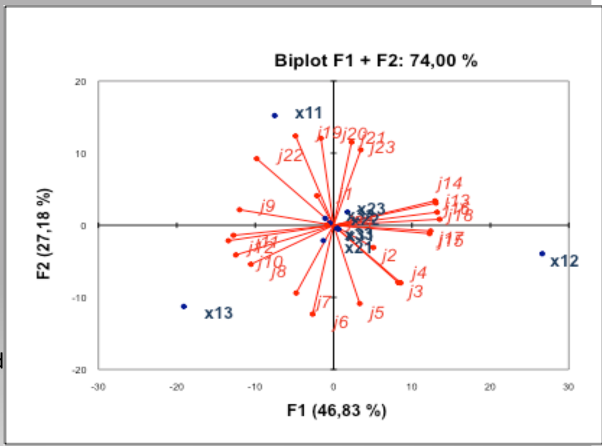
Judges 1-6 share common models and attract Judges 7-9

Judges migrate since they were initially embedded in less cohesive classes

PCA of Local difference models

Attribute-levels near the origine of the factorial plan are more sensible to the network effect

Attribute levels with higher coordinates explain the local difference:
Individual model vs/ neighbors mod



Some concluding remarks

Relational and attribute data

- to derive ad hoc relational data structures (affiliation and adjacency matrices)
- to enhance the interpretation of traditional network analysis from a different point of view:
    - i) the complementary use of valued graphs defined according to observed auxiliary information;
    - ii) the possibility to introduce explicative measures joining external information and relational data
    - iii) the interpretation of the results as complex data where groups of actors are defined and interpreted as "second order" individuals

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