Wave operators with non-Lipschitz coefficients: energy and observability estimates

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In the present talk we consider second order strictly hyperbolic operators with low regularity coefficients: namely,

$$Lu \;:=\; \partial_t^2 u \;-\; \sum_{j,k=1}^{N} \partial_j \Big(a_{jk}(t,x)\, \partial_k u \Big)\,,$$

where the a_{jk}'s are assumed to satisfy weaker conditions than the Lipschitz one.

In the first part, we will review basic facts about the well-posedness of the Cauchy problem related to L. In particular, we will explain the phenomenon of the *loss of derivatives* which occurs in the energy estimates due to the low regularity of the coefficients. Then, well-posedness of the corresponding Cauchy problem can be recovered just in the space H^∞, under suitable assumptions on the a_{jk}'s.

Nonetheless, despite to the global picture we have just described, we will also show a well-posedness result *without loss of derivatives* in the space $H^{1/2} \times H^{-1/2}$ for isotropic Zygmund continuous coefficients.

In the second part of the talk we will get interested in the control problem for L, in the special instance of space dimension $N = 1$ and for coefficients just depending on x.

By use of the technique of *sidewise energy estimates*, we will show how the low regularity of the coefficients affects also the observability properties, giving rise to an effect analogous to the loss of derivatives.

On the one hand, we will give a complete characterization of the relation between the modulus of continuity of the coefficients and the type of observability estimates (with finite, infinite or no loss) for the corresponding wave equation. On the other hand, we will see that Zygmund-type conditions escape such a classification and allow to go below the critical regularity.

References

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[3] F. Fanelli, E. Zuazua: *Weak observability estimates for* 1*-D wave equations with rough coefficients*, Ann. Inst. H. Poincaré Anal. Non Linéaire, to appear (2014).