MVA903

Corrigé du devoir n°2

Exercice 1

1°) On remarque que: $d(x^2+1) = 2x\,dx$ donc on a:

$$\frac{x}{x^2+1} = \frac{1}{2}\frac{(x^2+1)'}{x^2+1}$$

Dès lors $I_1 = \displaystyle\int_0^1 \frac{x}{x^2+1}\,dx = \frac{1}{2}\int_0^1 \frac{d(x^2+1)}{x^2+1} = \frac{1}{2}\Big[\ln|x^2+1|\Big]_0^1 = \frac{1}{2}\Big(\ln 2 - \ln 1\Big) = \frac{1}{2}\ln 2 = \ln(\sqrt{2})$

2°) $I_2 + I_1 = \displaystyle\int_0^1 \left(\frac{x^3}{x^2+1} + \frac{x}{x^2+1}\right)dx = \int_0^1 \left(\frac{x^3+x}{x^2+1}\right)dx = \int_0^1 \frac{x(x^2+1)}{(x^2+1)}\,dx = \int_0^1 x\,dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$;
or on a déjà calculer I_1.

3°) Dès lors, on a: $I_2 = (I_1+I_2) - I_1 = \dfrac{1}{2} - \ln\sqrt{2} = \dfrac{1}{2} - \dfrac{1}{2}\ln 2 = \dfrac{1}{2}\Big(1 - \ln 2\Big)$

Exercice 2

1°) On pose la division euclidienne classique du numérateur par le dénominateur:

$$\begin{array}{cccccccc|l}
 & & \cancel{x^3} & - & 3x^2 & + & x & & -x+1 \\ \cline{9-9}
- & (- & \cancel{x^3} & - & x^2) & & & & -x^2+2x+1 \\ \cline{2-5}
 & & & & \cancel{-2x^2} & + & x & & \\
 & & & - & (\cancel{-2x^2}) & + & 2x) & & \\ \cline{4-8}
 & & & & & & \cancel{-x} & & \\
 & & & & & - & (\cancel{-x} & + & 1) \\ \cline{5-8}
 & & & & & & & & -1
\end{array}$$

Donc on peut écrire $f(x) = -x^2 + 2x^2 + 1 - \dfrac{1}{-x+1} = -x^2 + 2x + 1 + \dfrac{1}{x-1}$ *ie* $a=-1,\ b=2;\ c=1,\ d=-1.$

2°) Sur $\left[0, \dfrac{1}{2}\right]$, $x \neq 1 \Rightarrow$ on peut écrire la décomposition en éléments simples vue en 1°)

$$\begin{aligned}
\Rightarrow\quad I &= \int_0^{\frac{1}{2}} \left(-x^2 + 2x + 1 + \frac{1}{x-1}\right)dx \\
&= \left[-\frac{x^3}{3} + x^2 + x + \ln|x-1|\right]_0^{\frac{1}{2}} \\
&= -\frac{1}{3}\frac{1}{8} + \frac{1}{4} + \frac{1}{2} + \ln\left(\frac{1}{2}\right) \\
&= -\frac{1}{24} + \frac{1}{4} + \frac{1}{2} - \ln 2 \\
&= -\frac{1}{24} + \frac{3}{4} - \ln 2 \\
&= \frac{17}{24} - \ln 2
\end{aligned}$$

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$\boxed{\textbf{Exercice } 3}$

1°) On fait la division euclidienne:

$$
\begin{array}{rrrrr|l}
 & \cancel{x^2} & & & -\ 1 & 2x-1 \\
\cline{6-6}
- & (\cancel{x^2} & - & \frac{x}{2}) & & \frac{x}{2}+\frac{1}{4} \\
\cline{2-5}
 & & & \cancel{\frac{x}{2}} & -\ 1 & \\
 & & - & (\cancel{\frac{x}{2}} & -\ \frac{1}{4}) & \\
\cline{3-5}
 & & & & -\ \frac{3}{4} &
\end{array}
$$

Donc on a: $f(x) = \frac{x}{2} + \frac{1}{4} - \frac{3}{4}\frac{1}{2x-1}$ ie $a = \frac{1}{2},\ b = \frac{1}{4},\ c = -\frac{3}{4}$

2°)

$$
\begin{aligned}
I = & \int_{-1}^{0} \left(\frac{x}{2} + \frac{1}{4} - \frac{3}{4}\frac{1}{2x-1} \right) dx \\
= & \ \frac{1}{2}\int_{-1}^{0} x_{,}dx + \frac{1}{4}\int_{-1}^{0} dx - \frac{3\cdot 1}{4\cdot 2}\int_{-1}^{0} \frac{2}{2x-1}\, dx \\
= & \ \frac{1}{2}\left[\frac{x^2}{2}\right]_{-1}^{0} + \frac{1}{4}\Big[x\Big]_{-1}^{0} - \frac{3}{8}\Big[\ln|2x-1|\Big]_{-1}^{0} \\
= & \ \frac{1}{4}\Big(0-1\Big) + \frac{1}{4}\Big(0-(-1)\Big) - \frac{3}{8}\Big(\ln 1 - \ln(|-2-1|)\Big) \\
= & \ -\cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \frac{3}{8}\Big(0 - \ln 3\Big) \\
= & \ \frac{3\ln 3}{8}
\end{aligned}
$$

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